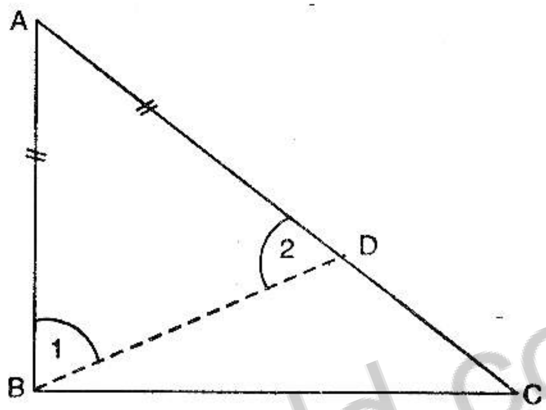
Unit 13

SIDES AND ANGLES OF A TRIANGLE

Theorem If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.



Given In ΔABC, $m\overline{AC} > m\overline{AB}$

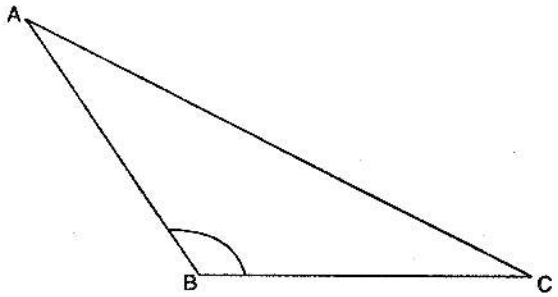
To Prove $m\angle ABC > m\angle ACB$

Construction On $\overline{AC}$ take a point D such that $\overline{AD} \cong \overline{AB}$ Join B to D so that ΔADB is an isosceles triangle. Label $\angle 1$ and $\angle 2$ as shown in the given figure.

Proof

Statements	Reasons
In ΔABD	
$m\angle 1 = m\angle 2$ ...(i)	Angle opposite to congruent sides, (construction)
In ΔBCD, $m\angle ACB < m\angle 2$	
i.e., $m\angle 2 > m\angle ACB$ ...(ii)	(An exterior angle of a triangle is greater than a non-adjacent interior angle).
$\therefore$ $m\angle 1 > m\angle ACB$ ....(iii)	By (i) and (ii)
But	
$m\angle ABC = m\angle 1 + m\angle DBC$	Postulate of addition of angles.
$\therefore$ $m\angle ABC > m\angle 1$ ......(iv)	
$\therefore$ $m\angle ABC > m\angle 1 > m\angle ACB$	
Hence $m\angle ABC > m\angle ACB$	By (iii) and (iv) (Transitive property of inequality of real number)

Example Prove that in a scalene triangle, the angle opposite to the largest side is of measure greater than 60°. (i.e., two-third of a right-angle).



Given In ΔABC, $m\overline{AC} > m\overline{AB}$, $m\overline{AC} > m\overline{BC}$.

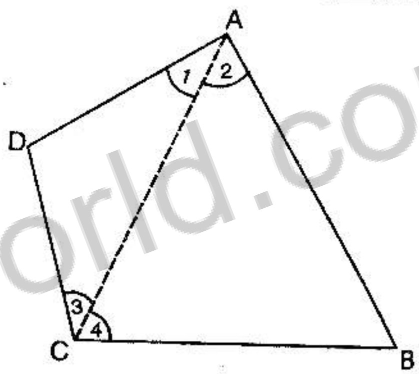
To Prove

$m\angle B > 60^\circ$.

Proof

Statements	Reasons
In ΔABC	
$m\angle B > m\angle C$	$m\overline{AC} > m\overline{AB}$ (given)
$m\angle B > m\angle A$	$m\overline{AC} > m\overline{BC}$ (given)
But $m\angle A + m\angle B + m\angle C = 180^\circ$	$\angle A, \angle B, \angle C$ are the angles of ΔABC
$\therefore$ $m\angle B + m\angle B + m\angle B > 180^\circ$	$m\angle B > m\angle C$, $m\angle B > m\angle A$ (proved)
Hence $m\angle B > 60^\circ$	$180^\circ/3 = 60^\circ$.

Example In a quadrilateral ABCD, $\overline{AB}$ is the longest side and $\overline{CD}$ is the shortest side. Prove that $m\angle BCD > m\angle BAD$.



Given In quad. ABCD, $\overline{AB}$ is the longest side and $\overline{CD}$ is the shortest side.

To Prove $m\angle BCD > m\angle BAD$

Construction

Join A to C.

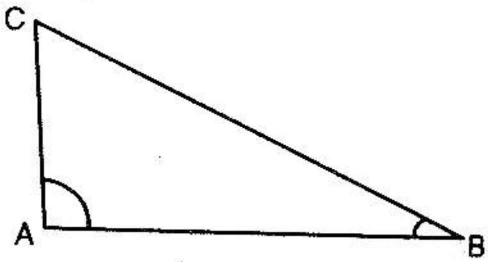
Name the angles $\angle 1, \angle 2, \angle 3$ and $\angle 4$ as shown in the figure.

Proof

Statements	Reasons
In ΔABC, $m\angle 4 > \angle 2$ ......(i)	$m\overline{AB} > m\overline{BC}$ (given)
In ΔACD, $m\angle 3 > m\angle 1$ ......(ii)	$m\overline{AD} > m\overline{CD}$ (given)
$\therefore$ $m\angle 4 + m\angle 3 > m\angle 2 + m\angle 1$	From I and II
Hence $m\angle BCD > m\angle BAD$	$\begin{cases} m\angle 4 + m\angle 3 = m\angle BCD \\ m\angle 2 + m\angle 1 = m\angle BAD \end{cases}$

Theorem:

If two angles of a triangle are unequal in measure, the side opposite to the greater angle is longer than the side opposite to the smaller angle.



Given In ΔABC, $m\angle A > m\angle B$

To Prove $m\overline{BC} > m\overline{AC}$

Proof

Statements	Reasons
If $m\overline{BC} \not> m\overline{AC}$, then	
either (i) $m\overline{BC} = m\overline{AC}$ or (ii) $m\overline{BC} < m\overline{AC}$	(Trichotomy property of real numbers)
From (i) if $m\overline{BC} = m\overline{AC}$, then $m\angle A = m\angle B$	(Angles opposite to congruent sides are congruent)
which is not possible	Contrary to the given
From (ii) if $m\overline{BC} < m\overline{AC}$, then $m\angle A < m\angle B$	(The angle opposite to longer side is greater than angle opposite to smaller side)
This is also not possible.	Contrary to the given
$\therefore$ $m\overline{BC} \neq m\overline{AC}$	
And $m\overline{BC} \not< m\overline{AC}$	
Thus $m\overline{BC} > m\overline{AC}$	Trichotomy property of real numbers.

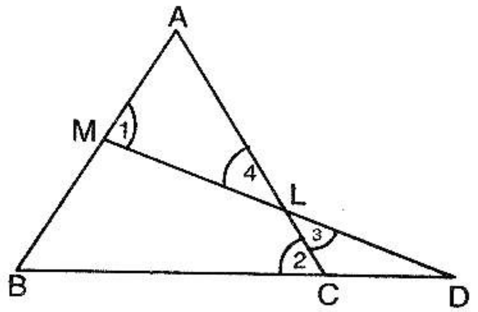
Note

(i) The hypotenuse of a right angle triangle is longer than each of the other two sides.

(ii) In an obtuse angled triangle, the side opposite to the obtuse angle is longer than each of the other two dies.

Example

ABC is an isosceles triangle with base $\overline{BC}$. On $\overrightarrow{BC}$ a point D is taken away from C. A line segment though D cuts $\overline{AC}$ at L and $\overline{AB}$ at M. Prove that $m\overline{AL} > m\overline{AM}$.



Given

In ΔABC, $\overline{AB} \cong \overline{AC}$.

D is a point on $\overrightarrow{BC}$ away from C.

A line segment through D cuts $\overline{AC}$ and L and $\overline{AB}$ at M.

To Prove

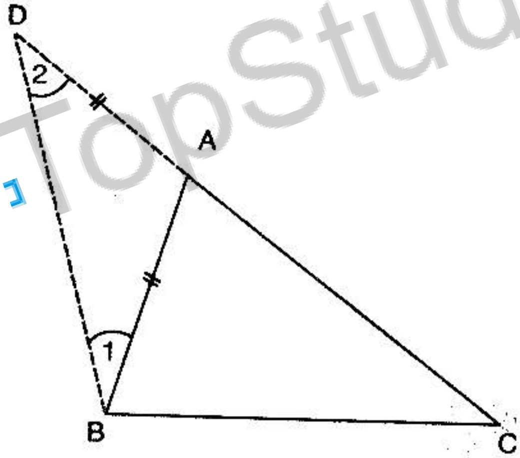
$m\overline{AL} > m\overline{AM}$

Proof

Statements		Reasons
In ΔABC		
$\angle B \cong \angle 2$	...I	$\overline{AB} \cong \overline{AC}$ (given)
In ΔMBD		
$m\angle 1 > m\angle B$	...II	($\angle 1$ is an ext. $\angle$ and $\angle B$ is its internal opposite $\angle$)
$\therefore$ $m\angle 1 > m\angle 2$	...III	From I and II
In ΔLCD,		
$m\angle 2 > m\angle 3$	IV	($\angle 2$ is an ext. $\angle$ and $\angle 3$ is its internal opposite $\angle$)
$\therefore$ $m\angle 1 > m\angle 3$	...V	From III and IV
But $\angle 3 \cong \angle 4$	...VI	Vertical angles
$\therefore$ $m\angle 1 > m\angle 4$		From V and VI
Hence $m\overline{AL} > m\overline{AM}$		In ΔALM, $m\angle 1 > m\angle 4$ (proved)

Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



To Prove

(i) $m\overline{AB} + m\overline{AC} > m\overline{BC}$

(ii) $m\overline{AB} + m\overline{BC} > m\overline{AC}$

(iii) $m\overline{BC} + m\overline{CA} > m\overline{AB}$

Construction

Take a point D on $\overline{CA}$ such that $\overline{AD} \cong \overline{AB}$. Join B to D and name the angles. $\angle 1$, $\angle 2$ as shown in the given figure.

Given

ΔABC

Proof

Statements		Reasons
In ΔABD,		
$\angle 1 \cong \angle 2$	...(i)	$\overline{AD} \cong \overline{AB}$ (construction)

Statements	Reasons
$m\angle DBC > m\angle 1$ ......(ii)	$m\angle DBC = m\angle 1 + m\angle ABC$
$\therefore$ $m\angle DBC > m\angle 2$ ......(iii)	From (i) and (ii)
In ΔDBC,	
$m\overline{CD} > m\overline{BC}$	By (iii)
i.e., $m\overline{AD} + m\overline{AC} > m\overline{BC}$	$m\overline{CD} = m\overline{AD} + m\overline{AC}$
Hence $m\overline{AB} + m\overline{AC} > m\overline{BC}$	$m\overline{AD} = m\overline{AB}$ (construction)
Similarly,	
$m\overline{AB} + m\overline{BC} > m\overline{AC}$	
And $m\overline{BC} + m\overline{CA} > m\overline{AB}$	

Example

Which of the following sets of lengths can be the lengths of the sides of a triangle.

(a) 2cm, 3cm, 5cm

(b) 3cm, 4cm, 5 cm

(c) 2cm, 4cm, 7cm

(a) $\therefore$ $2+3=5$

$\therefore$ This set of lengths cannot be those of the sides of a triangle.

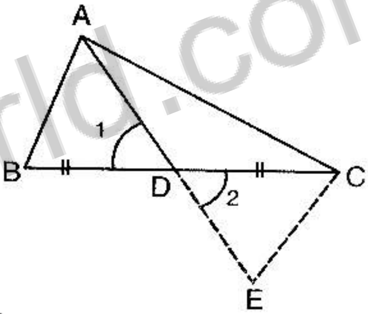
(b) $\therefore$ $3+4>5,\ 3+5>4,\ 4+5>3$

$\therefore$ This set can form a triangle.

(c) $\therefore$ $2+4<7$

$\therefore$ This set of lengths cannot be the sides of a triangle.

Example Prove that the sum of the measures of two sides of a triangle is greater than twice the measure of the median which bisects the third side.



Given

In ΔABC,

median AD bisects side $\overline{BC}$ at D.

To Prove

$m\overline{AB} + m\overline{AC} > 2m\overline{AD}$.

Construction On $\overrightarrow{AD}$, take a point E, such that $\overline{DE} \cong \overline{AD}$. Join C to E. Name the angles $\angle 1$, $\angle 2$ as shown in the figure.

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta ECD$	
$\overline{BD} \cong \overline{CD}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{AD} \cong \overline{ED}$	Construction
$\Delta ABD \cong \Delta ECD$	

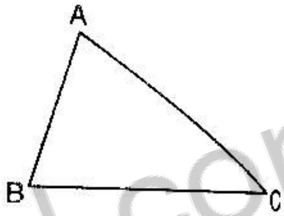
$\overline{AB} \cong \overline{EC}$ I	S.A.S. Postulate
$m\overline{AC}+m\overline{EC}>m\overline{AE}$ II	Corresponding sides of $\cong \Delta s$
$m\overline{AC}+m\overline{AB}>m\overline{AE}$	ACE is a triangle
Hence $m\overline{AC}+m\overline{AB}>2m\overline{AD}$	From I and II
	$m\overline{AE}=2m\overline{AD}$ (construction)

Example

Prove that the difference of measures of two sides of a triangle is less than the measure of the third side.

Given

ΔABC



To Prove

$m\overline{AC}-m\overline{AB}<m\overline{BC}$

$m\overline{BC}-m\overline{AB}<m\overline{AC}$

$m\overline{BC}-m\overline{AC}<m\overline{AB}$

Proof

Statements	Reasons
$m\overline{AB}+m\overline{BC}>m\overline{AC}$	ABC is a triangle
$(m\overline{AB}+m\overline{BC}-m\overline{AB})>(m\overline{AC}-m\overline{AB})$	Subtracting $m\overline{AB}$ from both sides.
$\therefore$ $m\overline{BC}>(m\overline{AC}-m\overline{AB})$	
Or $m\overline{AC}-m\overline{AB}<m\overline{BC}$ I	$a>b \Rightarrow b<a$
Similarly	
$m\overline{BC}-m\overline{AB}<m\overline{AC}$ $m\overline{BC}-m\overline{AC}<m\overline{AB}$	Reason similar to I

Exercise 13.1

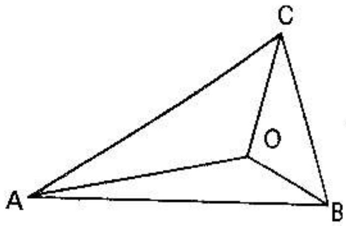
1. Two sides of a triangle measure 10 cm and 15 cm. Which of the following measure is possible for the third side?

(a) 5 cm (b) 20 cm
(c) 25 cm (d) 30 cm

Ans. 20cm.

2. O is an interior point of the ABC. Show that

$$m\overline{OA}+m\overline{OB}+m\overline{OC}>\frac{1}{2}(m\overline{AB}+m\overline{BC}+m\overline{CA})$$



Given: O is the interior point of ΔABC

To Prove:

$$m\overline{OA}+m\overline{OB}+m\overline{OC}>\frac{1}{2}\left(m\overline{AB}+m\overline{BC}+m\overline{CA}\right)$$

Construction:

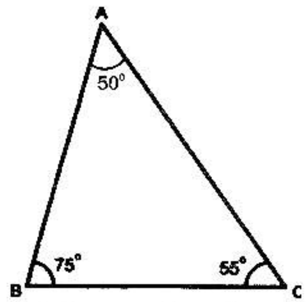
Join O with A, B and C.

Proof:

Statements	Reasons
ΔOAB	
$m\overline{OA}+m\overline{OB}>m\overline{AB}$ ....(i)	Sum of two sides > third side
Similarly	
$m\overline{OB}+m\overline{OC}>m\overline{BC}$ ....(ii)	Sum of two sides > third side
and	
$m\overline{OC}+m\overline{OA}>m\overline{CA}$ ....(iii)	
$2m\overline{OA}+2m\overline{OB}+2m\overline{OC}>m\overline{AB}+m\overline{BC}+m\overline{CA}$	Adding (i), (ii) and (iii)
$2\left(m\overline{OA}+m\overline{OB}+m\overline{OC}\right)>m\overline{AB}+m\overline{BC}+m\overline{CA}$	
$m\overline{OA}+m\overline{OB}+m\overline{OC}>\frac{1}{2}\left(m\overline{AB}+m\overline{BC}+m\overline{CA}\right)$	

3. In the ΔABC, $m\angle B = 75^\circ$ and $m\angle C = 55^\circ$. Which of the sides of the triangle is longest and which is the shortest?

Ans: Given a ΔABC in which



$m\angle B = 75^0$

$m\angle C = 55^0$

As $m\angle A + m\angle B + m\angle C = 180^0$

$m\angle A + 75^0 + 55^0 = 180^0$

$m\angle A + 130^0 = 180^0$

$m\angle A = 180^0 - 130^0$

$m\angle A = 50^0$

As we know in a triangle, the side opposite to greater angle is longer than the side opposite to smaller angle

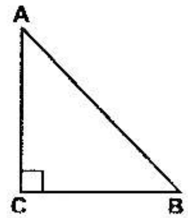
So $\quad m\overline{AC} > m\overline{BC}$

Hence longest side is $\overline{AC}$

and $\quad$ shortest side is $\overline{BC}$

4. Prove that in a right-angled triangle, the hypotenuse is longer than each of the other two sides.

Ans.



Given: ΔABC is a right angle triangle.
Hence AB is hypotenuse of ΔABC.

To prove:

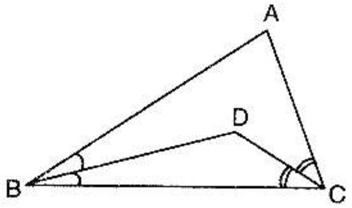
$mAB > mAC$ and $mAB > mBC$

Proof:

As ΔABC is a right angle triangle.
So $m\angle C = 90^0$ is the largest angle and the remaining angles $\angle A$ and $\angle B$ are acute.
So $m\angle C > m\angle A$ and $m\angle C > m\angle B$

As the side opposite to the greater angle is longer than the side opposite to the smaller angle.
Hence $mAB > mAC$ and $mAB > mBC$

5. In the triangular figure, $\overline{AB} > \overline{AC}$. $\overline{BD}$ and $\overline{CD}$ are the bisectors of $\angle B$ and $\angle C$ respectively. Prove that $\overline{BC} > \overline{DC}$.



Given: $\overline{AB} > \overline{BC}$, $\overline{BD}$ and $\overline{CD}$ are the bisectors of the angles B and C

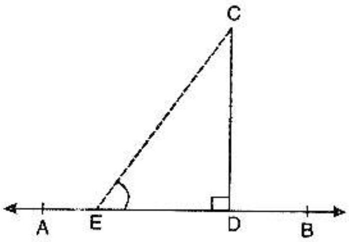
To Prove:

To prove = $\overline{BD} > \overline{CD}$

Proof

Statements	Reasons
∴ in ΔABC	
$\angle ACB > \angle ABC$	∴ $\overline{AB} > \overline{AC}$
$\frac{1}{2}\angle ACB > \frac{1}{2}\angle ABC$	
∴ $\angle BCD > \angle DBC$	$\overline{CD}, \overline{BD}$ are bisectors of $\angle C$, $\angle B$. The bigger sides is opposite the bigger angle
$\overline{BD} > \overline{CD}$	

Theorem From a point, outside a line, perpendicular is the shortest distance from the point to the line.



Given A line AB and a point C (not lying on $\overleftrightarrow{AB}$) and a point D on $\overleftrightarrow{AB}$ such that
$\overline{CD} \perp \overleftrightarrow{AB}$.

To Prove

$m\overline{CD}$ is the shortest distance from the point C to $\overleftrightarrow{AB}$.

Construction

Take a point E on $\overleftrightarrow{AB}$. Join C and E to form a ΔCDE

Proof:

Statements	Reasons
In ΔCDE	
$m\angle CDB > m\angle CED$	(An exterior angle of a triangle is greater

	than non adjacent interior angle).
But $m\angle CDB = m\angle CDE$	Supplement of right angle.
$\therefore$ $m\angle CDE > m\angle CED$	
or $m\angle CED < m\angle CDE$	$a > b \Rightarrow b < a$
or $m\overline{CD} < m\overline{CE}$	
But E is any point on AB	Side opposite to greater angle is greater.
Hence $m\overline{CD}$ is the shortest distance from C to $\overleftrightarrow{AB}$.	

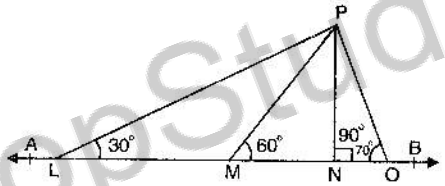
Note:

(i) The distance between a line and a point not on it, is the length of the perpendicular line segment from the point to the line.

(ii) The distance between a line and a point lying on it is zero

Exercise 13.2

1. In the figure, P is any point and AB is a line. Which of the following is the shortest distance between the point P and the line AB.



(a) $m\overline{PL}$ (b) $m\overline{PM}$

(c) $m\overline{PN}$ (d) $m\overline{PO}$

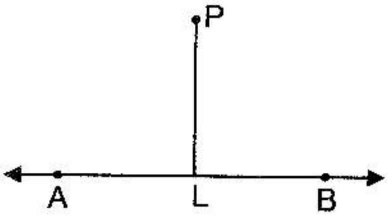
Ans. (c)

2. In the figure, P is any point lying away from the line AB. Then $m\overline{PL}$ will be the shortest distance if:

(a) $m\angle PLA = 80^\circ$

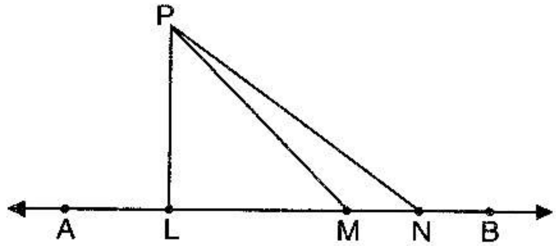
(b) $m\angle PLB = 100^\circ$

(c) $m\angle PLA = 90^\circ$



Ans. (c)

3. In the figure, $\overline{PL}$ is perpendicular to the line AB and $m\overline{LN} > m\overline{LM}$. Prove that $m\overline{PN} > m\overline{PM}$.



Ans. Here it is given $m\overline{PL}$ is perpendicular to line $\overleftrightarrow{AB}$ and $m\overline{LN} > m\overline{LM}$

Proof:

Here $m\overline{PN} > m\overline{PM}$

As $\overline{PL}$ is the shortest distance from P to line $\overleftrightarrow{AB}$. So

$\overline{PL} \perp \overleftrightarrow{AB}$

As we go away from point L, the distance from points to L increases Hence

$m\overline{PN} > m\overline{PM}$

4. Which of the following are true and which are false?

(i) The angle opposite to the longer side is greater. **TRUE**

(ii) In a right-angled triangle greater angle is of 60^o. **FALSE**

(iii) In an isosceles right-angled triangle, angles other than right angle are each of 45^o. **TRUE**

(iv) A triangle having two congruent sides is called equilateral triangle. FALSE

(v) A perpendicular from a point to t line is shortest distance. **TRUE**

(vi) Perpendicular to line form an angle of 90^o. **TRUE**

(vii) A point out-side the line is collinear. **FALSE**

(viii) Sum of two sides of triangle is greater than the third. **TRUE**

(ix) The distance between a line and a point on it is zero. **TRUE**

(x) Triangle can be formed of lengths 2 cm, 3 cm and 5 cm. **FALSE**

5. What will be angle for shortest distance from an outside point to the line?

Ans. 90^o

6. If 13 cm, 12 cm, and 5 cm are the lengths of a triangle, then verify that difference of measures of any two sides of a triangle is less than the measure of the third side.

Ans: (i) $13 - 12 = 1 < 15$

(ii) $12 - 4 = 7 < 13$

(iii) $13 - 5 = 8 < 12$

So verified

7. If 10 cm,6 cm and 8 cm are the lengths of a triangle, then verify that sum of measures of two sides of a triangle is greater than the third side.

Ans. (i) $10 + 6 = \; = 16 > 8$

(ii) $6 + 8 = 14 > 10$

(iii) $10+8 \; = 18 \; > 6$

8. 3 cm, 4 cm and 7 are not the lengths of the triangle. Give the reason.

Ans: $3 + 4 \not> 7$

9. If 3 cm and 4 cm are lengths of two sides of a right angle triangle then what should be the third length of the triangle.

Ans. Third length $= \sqrt{3^2 + 4^2}$

$= \sqrt{25} = 5cm$

OBJECTIVE

1. Which of the following sets of lengths can be the lengths of the sides of a triangle:

(a) 2cm, 3cm, 5cm

(b) 3cm, 4cm, 5cm

(c) 2cm, 4cm, 7cm

(d) None

2. Two sides of a triangle measure 10cm and 15cm. Which of the following measure is possible for the third side!

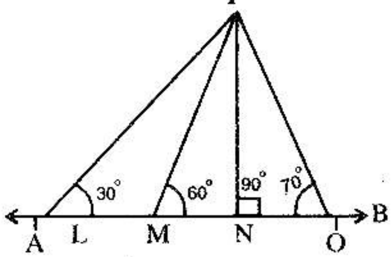
(a) 5cm

(b) 20cm

(c) 25cm

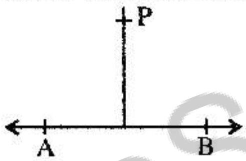
(d) 30cm

3. In the figure, P is any point and AB is a line. Which of the following is the short distance between the point P and line AB.



(a) $m\overline{PL}$
(b) $m\overline{PM}$
(c) $m\overline{PN}$
(d) $m\overline{PO}$

4. In the figure, P is any point lying away from the line AB. Then $m\overline{PL}$ will be shortest distance if:



(a) $m\angle PLA = 80^\circ$
(b) $m\angle PLB = 100^\circ$
(c) $m\angle PLA = 90^\circ$
(d) None

5. The angle opposite to the longer side is:
(a) Greater
(b) Shorter
(c) Equal
(d) None

6. In right angle triangle greater angle of:
(a) 60°
(b) 30°
(c) 75°
(d) 90°

7. In an isosceles right-angled triangle angles other than right angle are each of:
(a) 40°
(b) 45°
(c) 50°
(d) 55°

8. A triangle having two congruent sides is called ___ triangle.
(a) Equilateral
(b) Isosceles
(c) Right
(d) None

9. Perpendicular to line form an angle of ___
(a) 30°
(b) 60°
(c) 90°
(d) 120°

10. Sum of two sides of triangle is ___ than the third.
(a) Greater
(b) Smaller
(c) Equal
(d) None

11. The distance between a line and a point on it is ___
(a) Zero
(b) One
(c) Equal
(d) None

ANSWER KEY

1.	b	**2.**	b	**3.**	c	**4.**	c	**5.**	a
6.	d	**7.**	b	**8.**	a	**9.**	c	**10.**	a
11.	a								