Unit 03

LOGARITHMS

Scientific Notation

A number written in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer, is called the scientific notation.

Example

Write each of the following ordinary numbers in scientific notation

(i) 30600 (ii) 0.000058

Solution

(i) $30600 = 3.06 \times 10^4$

(move decimal point four places to the left)

(ii) $0.000058 = 5.8 \times 10^{-5}$

(move decimal point five places to the right)

Example

Change each of the following numbers from scientific notation to ordinary notation.

(i) 6.35×10^6 (ii) 7.61×10^{-4}

Solution

(i) $6.35 \times 10^6 = 6350000$

(move the decimal point six places to the right)

(ii) $7.61 \times 10^{-4} = 0.000761$

(move the decimal point four places to the left)

Exercise 3.1

Q1. Express each of the following numbers in scientific notation.

i) 5700

Sol: $5700 = 5.7 \times 10^3$ (move decimal point three places to left)

ii) 49,800,000

Sol: $49{,}800{,}000 = 4.98 \times 10^7$ (move decimal point seven places to left)

iii) 96,000,000

Sol: $96{,}000{,}000 = 9.6 \times 10^7$ (move decimal point seven places to left)

iv) 416.9

Sol: $416.9 = 4.169 \times 10^2$ (move decimal point two places to left)

v) 83,000

Sol: $83{,}000 = 8.3 \times 10^4$ (move decimal point four places to left)

vi) 0.00643

Sol: $0.00643 = 6.43 \times 10^{-3}$ (move decimal point three places to right)

vii) 0.0074

Sol: $0.0074 = 7.4 \times 10^{-3}$ (move decimal point three places to right)

viii) 60,000,000

Sol: $60{,}000{,}000 = 6.0 \times 10^7$ (move decimal point seven places to left)

ix) 0.00000000395

Sol: $0.00000000395 = 3.95 \times 10^{-9}$ (move decimal point nine places to right)

x) $\dfrac{275,000}{0.0025}$

Sol: $\dfrac{275,000}{0.0025}$

$=\dfrac{2.75\times10^{5}}{2.5\times10^{-3}}$ $\dfrac{\text{(move decimal point five places to left)}}{\text{(move decimal point three places to right)}}$

Q2. Express the following numbers in ordinary notation.

i) 6×10^{-4}

Sol: $6\times10^{-4} = 0.0006$ (move decimal point four places to left)

ii) 5.06×10^{10}

Sol: $5.06\times10^{10} = 50,600,000,000$ (move decimal point ten places to right)

iii) 9.018×10^{-6}

Sol: $9.018\times10^{-6} = 0.000009018$ (move decimal point six places to left)

iv) 7.865×10^{8}

Sol: $7.865\times10^{8} = 786,500,000$ (move decimal point eight places to right)

Logarithm of a Real Number

If $a^x = y$ then x is called the logarithm of y to the base 'a' and is written as $\log_a y = x$, where $a > 0, a \neq 1$ and $y > 0$

i.e., the logarithm of a number y to the base 'a' is the index x of the power to which a must be raised to get that number y.

The relations $a^x = y$ and $log_a y = x$ are equivalent. When one relation is given, it can be converted into the other. Thus

$$\boxed{a^x = y \Leftrightarrow \log_a y = x}$$

Example

Find $\log_4 2$, i.e., find log of 2 to the base 4.

Solution

Let $\log_4 2 = x$

Then its exponenetial form is $4^x = 2$

i.e., $2^{2x} = 2^1 \Rightarrow 2x = 1$

$\therefore \quad x = \dfrac{1}{2} \Rightarrow \log_4 2 = \dfrac{1}{2}$

Deductions from Definition of Logarithm

1. Since $a^0 = 1$, $\log_a 1 = 0$
2. Since $a^1 = a$, $\log_a a = 1$

Common Logarithm or Brigg's Logarithm

If the base of logarithm is taken as 10 then logarithm is called Common Logarithm.

Characteristic

The integral part of the logarithm of any number is called the characteristic.

Characteristic of Logarithm of a number > 1

The characteristic of the logarithm of a number greater than 1 is always one less than the number of digits in the integral part of the number.

When a number b is written in the scientific notation, i.e., in the form $b = a \times 10^n$ where $1 \leq a < 10$, the power of 10 i.e., n will give the characteristic of $\log b$.

Examples

Number	Scientific Notation	Characteristic of the Logarithm
1.02	1.02×10^0	0
99.6	9.96×10^1	1
102	1.092×10^2	2
1662.4	1.6624×10^3	3

Characteristic of Logarithm of a Number < 1

The characteristic of the logarithm of a number less than 1, is always negative and one more than the number of zeroes immediately after the decimal point of the number.

Example

Write the characteristic of the log of following numbers by expressing them in scientific notation and noting the power of 10.

0.872, 0.02, 0.00345

Number	Scientific Notation	Characteristic of the Logarithm
0.872	8.72×10^{-1}	-1
0.02	2.0×10^{-2}	-2
0.00345	3.45×10^{-3}	-3

Mantissa

The fractional part of the logarithm of a number is called the mantissa. Mantissa is always positive

Example

Find the mantissa of the logarithm of 43.254

Solution

Rounding off 43.254 we consider only the four significant digits 4325.

(i) We first locate the row corresponding to 43 in the log tables and

(ii) Proceed horizontally till we reach the column corresponding to 2. The number at the intersection is 6355.

(iii) Again proceeding horizontally till the mean difference column corresponding to 5 intersects this row, we get the number 5 at the intersection.

(iv) Adding the two numbers 6355 and 5 we get .6360 as the mantissa of the logarithm of 43.25.

Example

Find the mantissa of the logarithm of 0.002347

Solution

Here also, we consider only the four significant digits 2347

We first locate the row corresponding to 23 in the logarithm tables and proceed as before.

Along the same row to its intersection with the column corresponding to 4 the resulting number is 3692. The number at the intersection of this row and the mean difference column corresponding to 7 is 13. Hence the sum of 3692 and 13 gives the mantissa of the logarithm of 0.0023476 as 0.3705

Example

Find (i) log 278.23

(ii) log 0.07058

Solution

(i) 278.23 can be rounded off as 278.2

The characteristic is 2 and the mantissa, using log tables, is .4443

$\therefore$ log 278.23 = 2.4443

(ii) The characteristic of log 0.07058 is -2 which is written as $\bar{2}$ by convention.

Using log tables the mantissa is .8487, so that

$$\text{Log } 0.07058 = \bar{2}.8487$$

Example

Find the numbers whose logarithms are

(i) 1.3247 (ii) $\bar{2}.1324$

Solution

(i) 1.3247

Reading along the row corresponding to .32 (as mantissa = 0.3247), we get 2109 at the intersection of this row with the column corresponding to 4. The number at the intersection of this row and the mean difference column corresponding to 7 is 3. Adding 2109 and 3 we get 2112.

Since the characteristic is 1, it is increased by 1 (because there should be two digits in the integral part) and therefore the decimal point is fixed after two digits from left in 2112.

Hence antilog of 1.3247 is 21.12.

(ii) $\bar{2}.1324$

Proceeding as in (i) the significant figures corresponding to the mantissa 0.1324 are 1356. Since the characteristic is $\bar{2}$, its numerical value 2 is decreased by 1. Hence there will be one zero after the decimal point.

Hence antilog of $\bar{2}.1324$ is 0.01356.

Exercise 3.2

Q1. Find the common logarithm of the following numbers.

i) 232.92

232.92 can be rounded off as 232.9

Characteristic = 2

Mantissa = .3672

Hence log 232.92 = 2.3672

ii) 29.326

29.326 can be rounded off as 29.33

Characteristic = 1

Mantissa = .4673

Hence log 29.326 = 1.4673

iii) 0.00032

Characteristic = $\bar{4}$

Mantissa = .5051

Hence log 0.0032 = $\bar{4}.5051$

iv) 0.3206

Characteristic = $\bar{1}$

Mantissa = .5060

Hence log 0.3206 = $\bar{1}.5060$

Q2. If log 31.09 = 1.4926, find the values of following:

i) log 3.109

Sol: log 3.109

Characteristic = 0

Mantissa = .4926

So log 3.109 = 0.4926

ii) log 310.9

Sol: log 310.9

Characteristic = 2

Mantissa = .4926

So log 310.9 = 2.4926

iii) log 0.003109

Sol: log 0.003109

Characteristic = $\bar{3}$

Mantissa = .4926

So log 0.003109 = $\bar{3}.4926$

iv) log 0.3109

Sol: $\log 0.3109$

Characteristic $= \bar{1}$

Mantissa $= .4926$

So $\log 0.3109 = \bar{1}.4926$

Q3. Find the numbers whose common logarithms are:

i) 3.5621

let the number be x

$\log x = 3.5621$

Characteristic $= 3$

Mantissa $= .5621$

$x = \text{antilog } 3.5621 = 3648$

$x = 3648$

Hence 3648 is the required number

ii) $\bar{1}.7427$

Let the number be x

$\text{Log } x = \bar{1}.7427$

Characteristic $= \bar{1}$

Mantissa $= .7427$

$x = \text{antilog } \bar{1}.7427 = 0.5530$

$x = 0.5530$

Hence 0.5530 is the required number

Q4. What replacement for the unknown in each of following will make the statement true?

i) $\log_3 81 = L$

In exponential form

$$3^L = 81$$

$$3^L = 3^4$$

$\Rightarrow$ $\boxed{L = 4}$ Bases are equal so exponents are equal

ii) $\log_a 6 = 0.5$

In exponential form

$$a^{0.5} = 6$$

$$a^{\frac{1}{2}} = 6$$

Squaring both side

$$\left(a^{\frac{1}{2}}\right)^2 = (6)^2$$

$$\boxed{a = 36}$$

iii) $\log_5 n = 2$

In exponential form

$$5^2 = n$$

$\Rightarrow$ $\boxed{n = 25}$

iv) $10^P = 40$

In logarithmic form

$\text{Log}_{10} 40 = P$

or $\log 40 = P$

Characteristic $= 1$

Mantissa $= .6021$

So, $P = 1.6021$

Q5. Evaluate

i) $\log_2 \dfrac{1}{128}$

Let $x = \text{Log}_2 \dfrac{1}{128}$

In exponential form

$$2^x = \frac{1}{128}$$

$$2^x = \frac{1}{2^7}$$

$$2^x = 2^{-7}$$

$\Rightarrow$ $\boxed{x = -7}$

ii) log 512 to the base $2\sqrt{2}$

Sol: $\log_{2\sqrt{2}} 512$

Let $x = \log_{2\sqrt{2}} 512$

In exponential form

$$\left(2\sqrt{2}\right)^x = 512$$

$$\left(2\times 2^{\frac{1}{2}}\right)^x = 2^9$$

$$\left(2^{1+\frac{1}{2}}\right)^x = 2^9$$

$$\left(2^{\frac{3}{2}}\right)^x = 2^9$$

$$2^{\frac{3}{2}x} = 2^9$$

$$\Rightarrow \quad \frac{3}{2}x = 9$$

$$x = {}^3\cancel{9}\times\frac{2}{\cancel{3}}$$

$$\boxed{x=6}$$

Q6. Evaluate the value of 'x' from the following statements.

i) $\log_2 x = 5$

In exponential form

$$2^5 = x$$

$$\Rightarrow \quad \boxed{x=32}$$

ii) $\log_{81} 9 = x$

In exponential form

$$81^x = 9$$

$$\left(9^2\right)^x = 9$$

$$9^{2x} = 9^1$$

$$\Rightarrow \quad 2x = 1$$

$$\text{or} \quad \boxed{x=\frac{1}{2}}$$

iii) $\log_{64} 8 = \frac{x}{2}$

In exponential form

$$(64)^{\frac{x}{2}} = 8$$

$$\left(8^2\right)^{\frac{x}{2}} = 8$$

$$8^{\cancel{2}\times\frac{x}{\cancel{2}}} = 8$$

$$8^x = 8^1$$

$$\Rightarrow \quad \boxed{x=1}$$

iv) $\log_x 64 = 2$

In exponential form

$$x^2 = 64$$

$$x^2 = 8^2$$

$$\Rightarrow \quad \boxed{x=8}$$

v) $\log_3 x = 4$

In exponential form

$$3^4 = x$$

$$\Rightarrow \quad \boxed{x=81}$$

Laws of Logarithm

In this section we shall prove the laws of logarithm and then apply them to find products, quotients, powers and roots of numbers.

(i) $\log_a(mn) = \log_a m + \log_a n$

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

(iii) $\log_a m^n = n\log_a m$

(iv) $\log_a n = \log_b n \times \log_a b$

or $= \dfrac{\log_b n}{\log_b a}$

(i) $\log_a(mn) = \log_a m + \log_a n$:

Proof

Let $\log_a m = x$ *and* $\log_a n = y$

Writing in exponential form

$a^x = m$ *and* $a^y = n$

$\therefore \quad a^x \times a^y = mn$

i.e., $\quad a^{x+y} = mn$

or $\quad \log_a(mn) = x + y = \log_a m + \log_a n$

Hence $\quad \log_a(mn) = \log_a m + \log_a n$

Note

(i) $\log_a(mn) \neq \log_a m \times \log_a n$

(ii) $\log_a m + \log_a n \neq \log_a(m+n)$

(iii) $\log_a(mnp..) = \log_a m + \log_a n + \log_a p + ..$

The rule given above is useful in finding the product of two or more numbers using logarithms.

Example

Evaluate 291.3×42.36

Solution

$$
\begin{aligned}
\text{Let } x &= 291.3 \times 42.36 \\
\text{Then } \log x &= \log(291.3 \times 42.36) \\
&= \log 291.3 + \log 42.36 \\
&\quad (\log_a mn = \log_a m + \log_a n) \\
&= 2.4643 + 1.6269 = 4.0912 \\
x &= \text{antilog } 4.0912 = 12340
\end{aligned}
$$

Example

Evaluate 0.2913×0.004236.

Solution

$$
\begin{aligned}
\text{Let } y &= 0.2913 \times 0.004236 \\
\text{Then } \log y &= \log 0.2913 + \log 0.004236 \\
\log y &= \bar{1}.4643 + \bar{3}.6269 \\
\log y &= \bar{3}.0912 \\
y &= \text{anti log } \bar{3}.0912 \\
y &= 0.001234
\end{aligned}
$$

(ii) $\quad \log_a\left(\dfrac{m}{n}\right) = \log_a m - \log_a n$

Proof

Let $\quad \log_a m = x$ *and* $\log_a n = y$

So that $\quad a^x = m$ *and* $a^y = n$

$\therefore \quad \dfrac{a^x}{a^y} = \dfrac{m}{n} \Rightarrow a^{x-y} = \dfrac{m}{n}$

i.e.,

$\log_a\left(\dfrac{m}{n}\right) = x - y = \log_a m - \log_a n$

Hence $\log_a\left(\dfrac{m}{n}\right) = \log_a m - \log_a n$

Note

(i) $\log_a\left(\dfrac{m}{n}\right) \neq \dfrac{\log_a m}{\log_a n}$

(ii) $\log_a m - \log_a n \neq \log_a(m-n)$

(iii) $\log_a\left(\dfrac{1}{n}\right) = \log_a 1 - \log_a n = -\log_a n .....$

$(\because \log_a 1 = 0)$

Example

Evaluate $\dfrac{291.3}{42.36}$

Solution

Let $\quad x = \dfrac{291.3}{42.36}$ so that $\log x = \log \dfrac{291.3}{42.36}$

Then $\log x = \log 291.3 - \log 42.36$, ...

$\left(\log_a \dfrac{m}{n} = \log_a m - \log_a n\right)$

$\log x = 2.4643 - 1.6269 = 0.8374$

Thus $\quad x = \text{antilog } 0.8374 = 6.877$

Example

Evaluate $\dfrac{0.0002913}{0.04236}$

Solution

Let $\quad y = \dfrac{0.0002913}{0.04236}$ so that

$\log y = \log\left(\dfrac{0.002913}{0.04236}\right)$

then $\quad \log y = \log 0.002913 - \log 0.04236$

$$\log y = \bar{3}.4643 - \bar{2}.6269$$
$$= \bar{3} + (0.4643 - 0.6269) - \bar{2}$$
$$= \bar{3} - 0.1626 - \bar{2}$$
$$= \bar{3} + (1 - 0.1626) - 1 - \bar{2},$$
(adding and subtracting 1)
$$= \bar{2}.8374$$
$[\therefore\ \bar{3}-1-\bar{2} = -3-1-(-2) = -2 = \bar{2}]$

Therefore, $y = \text{antilog } \bar{2}.8374$

$y = 0.06877$

(iii) $\log_a(m^n) = n\log_a m$:

Proof

Let $\log_a m^n = x$, i.e., $a^x = m^n$

and $\log_a m = y$, i.e., $a^y = m$

Then $a^x = m^n = (a^y)^n$

i.e., $a^x = (a^y)^n = a^{yn} \Rightarrow x = ny$

i.e., $\log_a m^n = n\log_a m$

Example

Evaluate $\sqrt[4]{(0.0163)^3}$

Solution

Let $y = \sqrt[4]{(0.0163)^3} = (0.0163)^{3/4}$

$$\log y = \frac{3}{4}(\log 0.0163)$$
$$= \frac{3}{4} \times \bar{2}.2122$$
$$= \frac{\bar{6}.6366}{4}$$
$$= \frac{\bar{8}+2.6366}{4}$$
$$= \bar{2}+0.6592 = \bar{2}.6592$$

Hence $y = \text{antilog } \bar{2}.6592$

$= 0.04562$

(iv) Change of Base Formula:

$$\log_a n = \log_b n \times \log_a b \text{ or } \frac{\log_b n}{\log_b a}$$

Proof

Let $\log_b n = x$ so that $n = b^x$

Taking log to the base a, we have

$$\log_a n = \log_a b^x = x\log_a b = \log_b n \log_a b$$

Thus $\log_a n = \log_b n \log_a b$ ......(i)

Putting $n = a$ in the above result, we get

$$\log_b a \times \log_a b = \log_a a = 1$$

or $\log_a b = \dfrac{1}{\log_b a}$

Hence equation (i) gives

$$\log_a n = \frac{\log_b n}{\log_b a} \quad ......(ii)$$

Using the above rule, a natural logarithm can be converted to a common logarithm and vice versa.

$$\log_e n = \log_{10} n \times \log_e 10 \text{ or } \frac{\log_{10} n}{\log_{10} e}$$

$$\log_{10} n = \log_e n \times \log_{10} e \text{ or } \frac{\log_e n}{\log_e 10}$$

The values of $\log_e 10$ and $\log_{10} e$ are available from the tables:

$$\log_e 10 = \frac{1}{0.4343} = 2.3026 \quad \text{and}$$

$$\log_{10} e = \log 2.718 = 0.4343$$

Example

Calculate $\log_2 3 \times \log_3 8$

Solution

We know that

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\therefore \quad \log_2 3 \times \log_3 8 = \frac{\log 3}{\log 2} \times \frac{\log 8}{\log 3}$$

$$= \frac{\log 8}{\log 2} = \frac{\log 2^3}{\log 2}$$

$$= \frac{3\log 2}{\log 2} = 3$$

Exercise 3.3

Q1. Write the following into sum or difference.

i) $\log(A \times B)$

Sol: $\log(A \times B) = \log A + \log B$

ii) $\log \frac{15.2}{30.5}$

Sol: $\log \frac{15.2}{30.5} = \log 15.2 - \log 30.5$

iii) $\log \frac{21 \times 5}{8}$

Sol: $\log \frac{21 \times 5}{8} = \log 21 + \log 5 - \log 8$

iv) $\log \sqrt[3]{\frac{7}{15}}$

Sol: $\log \sqrt[3]{\frac{7}{15}} = \log \left(\frac{7}{15}\right)^{\frac{1}{3}} = \frac{1}{3} \log \left(\frac{7}{15}\right)$

$$= \frac{1}{3}(\log 7 - \log 15)$$

v) $\log \frac{(22)^{\frac{1}{3}}}{5^3}$

Sol: $\log \frac{(22)^{\frac{1}{3}}}{5^3} = \log (22)^{\frac{1}{3}} - \log 5^3$

$$= \frac{1}{3} \log 22 - 3 \log 5$$

vi) $\log \frac{25 \times 47}{29}$

$$= \log 25 + \log 47 - \log 29$$

Q2. Express

$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$

as a single logarithm

Sol:

$\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$

$= \log x - \log x^2 + \log(x+1)^3 - \log(x^2-1)$

$= \log x + \log(x+1)^3 - \log x^2 - \log(x^2-1)$

$$= \log \frac{x(x+1)^3}{x^2(x^2-1)}$$

$$= \log \frac{(x+1)^3}{x(x-1)(x+1)}$$

$$= \log \frac{(x+1)^2}{x(x-1)}$$

Q3. Write the following in the form of a single logarithm.

i) $\log 21 + \log 5$

Sol: $\log 21 + \log 5$

$= \log 21 \times 5$

ii) $\log 25 - 2\log 3$

$= \log 25 - \log 3^2$

$= \log \frac{25}{3^2} = \log \frac{25}{9}$

iii) $2\log x - 3\log y$

Sol: $2\log x - 3\log y$

$= \log x^2 - \log y^3$

$= \log \frac{x^2}{y^3}$

iv) $\log 5 + \log 6 - \log 2$

Sol: $\log 5 + \log 6 - \log 2$

$= \log \frac{5 \times 6}{2}$

Q4. Calculate the following:

i) $\log_3 2\times\log_2 81$

Sol: As we know that $\log_a n=\frac{\log_b n}{\log_b a}$

$$\therefore \log_3 2\times\log_2 81=\frac{\log 2}{\log 3}\times\frac{\log 81}{\log 2}$$

$$=\frac{\log 81}{\log 3}$$

$$=\frac{\log 3^4}{\log 3}$$

$$=\frac{4\log 3}{\log 3}$$

$$=4$$

ii) $\log_5 3\times\log_3 25$

Sol: As we know that

$$\log_a n=\frac{\log_b n}{\log_b a}$$

$$\log_5 3\times\log_3 25=\frac{\cancel{\log 3}}{\log 5}\times\frac{\log 25}{\log 3}$$

$$=\frac{\log 25}{\log 5}$$

$$=\frac{\log 5^2}{\log 5}$$

$$=\frac{2\cancel{\log 5}}{\cancel{\log 5}}$$

$$=2$$

Q5. If $\log 2=0.3010$, $\log 3=0.4771$, $\log 5=0.6990$**, then find the values of the following.**

i) $\log 32$

Sol: $\log 32$

$$=\log 2^5$$

$$=5\log 2$$

$$=5(0.3010)$$

$$=1.5050$$

ii) $\log 24$

$$=\log 8\times 3$$

$$=\log 2^3\times 3$$

$$=\log 2^3+\log 3$$

$$=3\log 2+\log 3$$

$$=3(0.3010)+0.4771$$

$$=0.9030+0.4771$$

$$=1.3801$$

iii) $\log\sqrt{3\frac{1}{3}}$

$$=\log\sqrt{\frac{10}{3}}$$

$$=\log\left(\frac{2\times 5}{3}\right)^{\frac{1}{2}}$$

$$=\frac{1}{2}\log\left(\frac{2\times 5}{3}\right)=\frac{1}{2}(\log 2+\log 5-\log 3)$$

$$=\frac{1}{2}(0.3010+0.6990-0.4771)$$

$$=\frac{1}{2}(0.5229)$$

$$=0.2615$$

iv) $\log\frac{8}{3}$

$$=\log\frac{2^3}{3}$$

$$=\log 2^3-\log 3$$

$$=3\log 2-\log 3$$

$$=3(0.3010)-0.4771$$

$$=0.4259$$

v) $\log 30$

$$=\log 2\times 3\times 5$$

$$=\log 2+\log 3+\log 5$$

$$=0.3010+0.4771+0.6990$$

$$=1.4771$$

Applications of logarithm

Example

Show that

$$7\log\frac{16}{15}+5\log\frac{25}{24}+\log\frac{81}{80}=\log 2.$$

Solution

L.H.S $=7\log\frac{16}{15}+5\log\frac{25}{24}+\log\frac{81}{80}$

$= \quad 7[\log 16-\log 15]+5[\log 25-\log 24]+3[\log 81-\log 80]$

$= \quad 7[\log 2^4-\log(3\times5)]+5[\log 5^2-\log(2^3\times3)]+3[\log 3^4-\log(2^4\times5)]$

$=7[4\log 2-\log 3-\log 5]+5[2\log 5-3\log 2-\log 3]+3[4\log 3-4\log 2-\log 5]$

$= \quad (28-15-12)\log 2+(-7-5+12)\log 3+(-7+10-3)\log 5$

$= \quad \log 2+0+0=\log 2$ = R.H.S

Example

Evaluate:

$$\sqrt[3]{\frac{0.07921\times(18.99)^2}{(5.79)^4\times0.9474}}$$

Let y $=$

$$\sqrt[3]{\frac{0.07921\times(18.99)^2}{(5.79)^4\times0.9474}} = \left(\frac{0.07921\times(18.99)^2}{(5.79)^4\times0.9474}\right)^{1/3}$$

Log y $=$

$$\frac{1}{3}\log\left(\frac{0.07921\times(18.99)^2}{(5.79)^4\times0.9474}\right)$$

$$=\frac{1}{3}[\log\{0.07921\times(18.99)^2\}-\log\{(5.79)^4\times0.9474\}]$$

$$=\frac{1}{3}[\log 0.07921+2\log 18.99-4\log 5.79-\log 0.9474\}$$

$$=\frac{1}{3}[\bar{2}.8988+2(1.2786)-4(0.7627)-\bar{1}.9765]$$

$$=\frac{1}{3}[\bar{2}.8988+2.5572-3.0508-\bar{1}.9765]$$

$$=\frac{1}{3}[-2+0.8988+2.5572-3.0508+1-0.9765]$$

$$=\frac{1}{3}(\bar{2}.4287)$$

$$=\frac{1}{3}(\bar{3}+1.4287)$$

$$=\bar{1}+0.4762=\bar{1}.4762$$

y $=$ antilog $\bar{1}.4762=0.2993$

Example

Given $A = A_0e^{-kd}$. If k = 2, what should be the value of d to make $A=\frac{A_0}{2}$?

Solution

Given that $A = A_0e^{-kd}$ $\Rightarrow$

$$\frac{A}{A_0}=e^{-kd}$$

Substituting k = 2 and $A=\frac{A_0}{2}$,

we get $\frac{1}{2}=e^{-2d}$

Taking common log on both sides,

$\log_{10}1-\log_{10}2=-2d\log_{10}e$,

where $e = 2.718$

$0-0.3010=-2d\,(0.4343)$

$$d=\frac{0.3010}{2\times0.4343}=0.3465$$

Exercise 3.4

Q1. Use log tables to find the values of

i) 0.8176×13.64

Sol: Let $x = 0.8176 \times 13.64$

Taking log of both sides

$\log x = \log 0.8176 \times 13.64$

$\log x = \log 0.8176 + \log 13.64$

$= \bar{1}.9125 + 1.1348$

$= -1 + 0.9125 + 1.1348$

$\log x = 1.0473$

Characteristics = 1

Mantissa = .0473

x = antilog 1.0473 = 11.15

ii) $(789.5)^{\frac{1}{8}}$

Sol: Let $x = (789.5)^{\frac{1}{8}}$

Taking log of both sides

$\log x = \log(789.5)^{\frac{1}{8}}$

$= \frac{1}{8}\log(789.5)$

$= \frac{1}{8}(2.8974)$

$\log x = 0.3622$

Characteristics = 0

Mantissa = .3622

x = antilog 0.3622 = 2.302

iii) $\dfrac{0.678 \times 9.01}{0.0234}$

Let $x = \dfrac{0.678 \times 9.01}{0.0234}$

Taking log of both sides

$\log x = \log \dfrac{0.678 \times 9.01}{0.0234}$

$= \log 0.678 + \log 9.01 - \log 0.0234$

$= \bar{1}.8312 + 0.9547 - (\bar{2}.3692)$

$= -1 + 0.8312 + 0.9547 - (-2 + 0.3692)$

$= -1 + 0.8312 + 0.9547 + 2 - 0.3692$

$\log x = 2.4167$

Characteristics = 2

Mantissa = .4167

x = antilog 2.4167 = 261.0

iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Sol: Let $x = \sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Taking log of both sides

$\log x = \log(2.709)^{\frac{1}{5}} \times (1.239)^{\frac{1}{7}}$

$= \log(2.709)^{\frac{1}{5}} + \log(1.239)^{\frac{1}{7}}$

$= \frac{1}{5}\log(2.709) + \frac{1}{7}\log(1.239)$

$= \frac{1}{5}(0.4328) + \frac{1}{7}(0.0931)$

$= 0.0866 + 0.0133$

$\log x = 0.0999$

Characteristics = 0

Mantissa = .0999

$x = $ antilog 0.0999

$x = 1.259$

v) $\dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

Sol: Let $x = \dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

Taking log of both sides

$\log x = \log \dfrac{(1.23)(0.6975)}{(0.0075)(1278)}$

$= \log 1.23 + \log 0.6975 - \log 0.0075 - \log 1278$

$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$

$= 0.0899 - 1 + 0.8435 + 3 - 0.8751 - 3.1065$

$$\log x = -1.0482$$
$$= -2 + 2 - 1.0482$$
$$= -2 + 0.9518$$
$$\log x = \bar{2}.9518$$

Characteristics = $\bar{2}$

Mantissa = .9518

$x =$ antilog $\bar{2}.9518$ = 0.0895

vi) $\sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

Let $x = \sqrt[3]{\dfrac{0.7214 \times 20.37}{60.8}}$

$$x = \left(\frac{0.7214 \times 20.37}{60.8}\right)^{\frac{1}{3}}$$

Taking log of both sides

$$\log x = \log\left(\frac{0.7214 \times 20.37}{60.8}\right)^{\frac{1}{3}}$$
$$= \frac{1}{3}\log\left(\frac{0.7214 \times 20.37}{60.8}\right)$$
$$= \frac{1}{3}(\log 0.7214 + \log 20.37 - \log 60.8)$$
$$= \frac{1}{3}(\bar{1}.8582 + 1.3090 - 1.7839)$$
$$= \frac{1}{3}(-1 + 0.8582 + 1.3090 - 1.7839)$$
$$= \frac{1}{3}(-0.6167)$$
$$\log x = -0.2056$$
$$= -1 + 1 - 0.2056$$
$$= -1 + 0.7944$$
$$\log x = \bar{1}.7944$$

Characteristics = $\bar{1}$

Mantissa = .7944

$x =$ antilog $\bar{1}.7944$

= 0.6229

vii) $\dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

Sol: Let $x = \dfrac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

$$x = \frac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$$

Taking log of both sides

$$\log x = \log \frac{83 \times (92)^{\frac{1}{3}}}{127 \times (246)^{\frac{1}{5}}}$$
$$= \log 83 + \log(92)^{\frac{1}{3}} - \log 127 - \log(246)^{\frac{1}{5}}$$
$$= \log 83 + \frac{1}{3}\log(92) - \log 127 - \frac{1}{5}\log(246)$$
$$= 1.9191 + \frac{1}{3}(1.9638) - 2.1038 - \frac{1}{5}(2.391)$$
$$= 1.9191 + 0.6546 - 2.1038 - 0.4782$$
$$\log x = -0.0083$$
$$= -1 + 1 - 0.0083$$
$$= -1 + 0.9917$$
$$\log x = \bar{1}.9917$$

Characteristics = $\bar{1}$

Mantissa = .9917

$x =$ antilog $\bar{1}.9917$ = 0.9811

viii) $\dfrac{(438)^3 \sqrt{0.056}}{(388)^4}$

Sol: Let $x = \dfrac{(438)^3 \sqrt{0.056}}{(388)^4}$

$$x = \frac{(438)^3 \times (0.056)^{\frac{1}{2}}}{(388)^4}$$

Taking log of both sides

$$\log x = \log \frac{(438)^3 \times (0.056)^{\frac{1}{2}}}{(388)^4}$$

$$= \log(438)^3 + \log(0.056)^{\frac{1}{2}} - \log(388)^4$$

$$= 3\log(438) + \frac{1}{2}\log(0.056) - 4\log(388)$$

$$= 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$$

$$= 3(2.6415) + \frac{1}{2}(-2 + 0.7482) - 4(2.5888)$$

$$= 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$$

$$= 7.9245 - 0.6259 - 10.3552$$

$$\log x = -3.0566$$

$$= -4 + 4 - 3.0566$$

$$= -4 + 0.9434$$

$$\log x = \bar{4}.9434$$

Characteristic = $\bar{4}$

Mantissa = .9434

$x =$ antilog $\bar{4}.9434$ = 0.0008778

Q2. A gas is expanding according to the law $PV^n = C$. Find C when P=80, V=3.1 and $n = \frac{5}{4}$.

Sol: $PV^n = C$

Taking log of both sides:

$$\log PV^n = \log C$$

$$\log P + \log V^n = \log C$$

$$\log C = \log P + n \log V$$

Putting $P = 80$, $V = 3.1$ and $n = \frac{5}{4}$

$$\log C = \log 80 + \frac{5}{4}\log 3.1$$

$$= 1.9031 + \frac{5}{4}(0.4914)$$

$$= 1.9031 + 0.6143$$

$$\log C = 2.5174$$

Characteristic = 2

Mantissa = .5174

C = antilog 2.5174

C = 329.2 unit

Q3. The formula $p = 90(5)^{-\frac{q}{10}}$ applies to the demand of a product, where 'q' is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs. 18.00?

Sol: $p = 90(5)^{-\frac{q}{10}}$

$q = ?$ and $p = Rs.\,18.00$

As $p = 90(5)^{-\frac{q}{10}}$

$$18 = 90(5)^{\frac{-q}{10}}$$

Taking log of both sides

$$\log 18 = \log 90(5)^{-\frac{q}{10}}$$

$$\log 18 = \log 90 + \log(5)^{-\frac{q}{10}}$$

$$\log 18 - \log 90 = \frac{-q}{10}\log 5$$

$$10(\log 18 - \log 90) = -q \log 5$$

$$10(1.2553 - 1.9542) = -q(0.6990)$$

$$-6.989 = -q(0.6990)$$

$$\Rightarrow \quad q(0.6990) = 6.989$$

$$q = \frac{6.989}{0.6990}$$

$$q = 9.998$$

$q = 10$ approximately

So 10 units will be demanded

OR

$$p = 90\ (5)^{-q/10}$$

Taking log of both sides

$$\log p = \log 90\ (5)^{-q/10}$$

$$\log p = \log 90 + \log\ (5)^{-q/10}$$

$$\log p = \log 90 - \frac{q}{10}\ \log 5$$

$$\frac{q}{10}\ \log 5 = \log 90 - \log p$$

$$\frac{q}{10}\ \log 5 = \log 90 - \log 18$$

$$\frac{q}{10}\ \log 5 = \log\ \frac{90}{18}$$

$$\frac{q}{10}\ \log 5 = \log\ 5$$

$$\frac{q}{10} = \frac{\log 5}{\log 5}$$

$$\frac{q}{10} = 1$$

q = 10 Units

Q4. **If** $A = \pi r^2$

$$\pi = \frac{22}{7},\ r = 15,\ A = ?$$

As $\quad A = \pi r^2$

Taking log of both sides

$$\log A = \log \pi r^2$$

$$= \log \pi + \log r^2$$

$$= \log \pi + 2\log r$$

$$= \log \frac{22}{7} + 2\log 15$$

$$= \log 22 - \log 7 + 2\log 15$$

$$= 1.3424 - 0.8451 + 2(1.1761)$$

$$= 1.3424 - 0.8451 + 2.3522$$

$$\log A = 2.8495$$

Characteristics = 2

Mantissa = .8495

A = antilog 2.8495

A = 707.1

Q5. **If** $v = \frac{1}{3}\pi r^2 h$, **find** v **when** $\pi = \frac{22}{7}, r = 2.5$ **and** $h = 4.2$

Sol: $\quad v = \frac{1}{3}\pi r^2 h$

$\pi = \frac{22}{7},\ r = 2.5$ and $h = 4.2,\ v = ?$

As $v = \frac{1}{3}\pi r^2 h$

Taking log of both sides

$$\log v = \log \frac{1}{3}\pi r^2 h$$

$$= \log \frac{1}{3} + \log \pi + \log r^2 + \log h$$

$$= \log \frac{1}{3} + \log \frac{22}{7} + 2\log r + \log h$$

$$= \log 1 - \log 3 + \log 22 - \log 7 + 2\log 2.5 + \log 4.2$$

$$= 0 - 0.4771 + 1.3424 - 0.8451 + 2(0.3979) + 0.6232$$

$$\log v = 1.4392$$

Characteristics = 1

Mantissa = .4392

$v =$ antilog 1.4392

$v =$ 27.49

Review Exercise 3

Q3. Find the value of 'x' in the following.

i) $\log_3 x = 5$

Sol. $\log_3 x = 5$

In exponential form

$x = 3^5$

$\Rightarrow$ $x = 243$

ii) $\log_4 256 = x$

Sol. $\log_4 256 = x$

In exponential form

$4^x = 256$

$4^x = 4^4$

$\Rightarrow$ $x = 4$

iii) $\log_{625} 5 = \frac{1}{4}x$

Sol. $\log_{625} 5 = \frac{1}{4}x$

In exponential form

$(625)^{\frac{1}{4}x} = 5$

$(5^4)^{\frac{1}{4}x} = 5$

$5^{4\times\frac{1}{4}x} = 5$

$5^x = 5^1$

$\Rightarrow$ $x = 1$

iv) $\log_{64} x = -\frac{2}{3}$

Sol. $\log_{64} x = -\frac{2}{3}$

In exponential form

$x = 64^{\frac{-2}{3}}$

$x = (4^3)^{\frac{-2}{3}}$

$= 4^{\cancel{3}\left(-\frac{2}{\cancel{3}}\right)}$

$x = 4^{-2}$

$x = \frac{1}{4^2}$

$x = \frac{1}{16}$

Q4. Find the value of 'x' in the following.

i) $\log x = 2.4543$

Characteristic = 2

Mantissa = .4543

x = antilog 2.4543

= 284.6

ii) $\log x = 0.1821$

Characteristic = 0

Mantissa = .1821

x = antilog 0.1821

= 1.521

iii) $\log x = 0.0044$

Characteristic = 0

Mantissa = .0044

x = antilog 0.0044

x = 1.010

iv) $\log x = \bar{1}.6238$

Characteristic = $\bar{1}$

Mantissa = .6238

x = antilog $\bar{1}.6238$

x = 0.4205

Q5. If log2 = 0.3010, log3 = 0.4771 and log 5 = 0.6990, then find the values of the following.

i) log45

Sol. log45

$= \log 3^2 \times 5$

$= \log 3^2 + \log 5$

$= 2\log 3 + \log 5$

$= 2(0.4771) + 0.6990$

$= 0.9542 + 0.6990$

$= 1.6532$

ii) $\log \frac{16}{15}$

$= \log \frac{2^4}{3 \times 5}$

$= \log 2^4 - \log 3 - \log 5$

$= 4\log 2 - \log 3 - \log 5$

$= 4(0.3010) - 0.4771 - 0.6990$

$= 1.2040 - 0.4771 - 0.6990$

$= 0.0279$

iii) log 0.048

$= \log \frac{48}{1000}$

$= \log \frac{16 \times 3}{10^3}$

$= \log \frac{2^4 \times 3}{2^3 \times 5^3}$

$= \log \frac{2 \times 3}{5^3}$

$= \log 2 + \log 3 - \log 5^3$

$= \log 2 + \log 3 - 3\log 5$

$= 0.3010 + 0.4771 - 3(0.6990)$

$= -1.3189$

$= -2 + 2 - 1.3189$

$= -2 + 0.6811$

$= \bar{2}.6811$

Q6. Simplify the following:

i) $\sqrt[3]{25.47}$

Sol. Let $x = (25.47)^{\frac{1}{3}}$

Taking log of both sides

$\log x = \log (25.47)^{\frac{1}{3}}$

$= \frac{1}{3}\log(25.47)$

$= \frac{1}{3}(1.4060)$

$\log x = 0.4687$

Characteristic = 0

Mantissa = .4687

x = antilog 0.4687

x = 2.942

ii) $\sqrt[5]{342.2}$

Sol. Let $x = (342.2)^{\frac{1}{5}}$

Taking log of both sides

$\text{Log } x = \log (342.2)^{\frac{1}{5}}$

$= \frac{1}{5}\log(342.2)$

$= \frac{1}{5}(2.5343)$

$\log x = 0.5069$

Characteristic = 0

Mantissa = .5069

x = antilog 0.5069

x = 3.213

iii) $\frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$

Sol: Let $x = \frac{(8.97)^3 \times (3.95)^2}{(15.37)^{\frac{1}{3}}}$

Taking log of both sides

$\log x = \log \frac{(8.97)^3 \times (3.95)^2}{(15.37)^{\frac{1}{3}}}$

$$= \log(8.97)^3 + \log(3.95)^2 - \log(15.37)^{\frac{1}{3}}$$

$$= 3\log(8.97) + 2\log(3.95) - \frac{1}{3}\log(15.37)$$

$$= 3(0.9528) + 2(0.5966) - \frac{1}{3}(1.1867)$$

$$= 2.8584 + 1.1932 - 0.3956$$

$\log x = 3.6560$

Characteristic = 3

Mantissa = .6560

x = antilog 3.6560

x = 4529

Objective

1. If $a^x = n$, then _____

(a) $a = \log_x n$ (b) $x = \log_n a$

(c) $x = \log_a n$ (d) $a = \log_n x$

2. The relation of $y = \log_z x$ implies

(a) $x^y = z$ (b) $z^y = x$

(c) $x^z = y$ (d) $y^z = x$

3. The logarithm of unity to any base is ___

(a) 1 (b) 10

(c) e (d) 0

4. The logarithm of any number to itself as base is___

(a) 1 (b) 0

(c) −1 (d) 10

5. log e = ____ where e ≈ 2. 718

(a) 0 (b) 0.4343

(c) ∞ (d) 1

6. The value of $\log\left(\frac{p}{q}\right)$ is ___

(a) log p −log q

(b) $\frac{\log p}{\log q}$

(c) log p + log q

(d) log q − log p

7. $\log m^n$ can be written as

(a) $(\log m)^n$ (b) m log n

(c) n log m (d) log (mn)

8. $\log_b a \times \log_c b$ can be written as___

(a) $\log_c a$ (b) $\log_a c$

(c) $\log_a b$ (d) $\log_b c$

9. $\text{Log}_y\, x$ will be equal to___

(a) $\frac{\log_z x}{\log_y z}$ (b) $\frac{\log_x z}{\log_y z}$

(c) $\frac{\log_z x}{\log_z y}$ (d) $\frac{\log_z y}{\log_z x}$

10. For common logarithm, the base is____

(a) 2 (b) 10

(c) e (d) 1

11. For natural logarithm, the base is___

(a) 10 (b) e

(c) 2 (d) 1

12. The integral part of the common logarithm of a number is called the___

(a) Characteristic (b) Mantissa

(c) Logarithm (d) None

13. The decimal part of the common logarithm of a number is called the _____:

(a) Characteristic (b) Mantissa

(c) Logarithm (d) None

14. If x = log y, then y is called the ___ of x.
(a) Antilogarithm (b) Logarithm
(c) Characteristic (d) None

15. If the characteristic of the logarithm of a number is $\bar{2}$, that number will have zero (s) immediately after the decimal point.
(a) One (b) Two
(c) Three (d) Four

16. If the characteristic of the logarithm of a number is 1, that number will have ____ digits in its integral part
(a) 2
(b) 3
(c) 4
(d) 5

17. The value of x in $\log_3 x = 5$ is_____
(a) 243 (b) 143
(c) 200 (d) 144

18. The value of x in log x = 2.4543 is
(a) 284.6 (b) 1.521
(c) 1.1010 (d) 0.4058

19. The number corresponding to a given logarithm is known as ___
(a) Logarithm (b) Antilogarithm
(c) Characteristic (d) None

20. 30600 in scientific notation is ___
(a) 3.06×10^4 (b) 3.006×10^4
(c) 30.6×10^4 (d) 306×10^4

21. 6.35×10^6 in ordinary notation is___
(a) 6350000 (b) 635000
(c) 6350 (d) 63500

22. A number written in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer is called ____
(a) Scientific notation
(b) Ordinary notation
(c) Logarithm notation
(d) None

23. log p – log q is same as
(a) $\log\left(\frac{q}{p}\right)$
(b) $\log(p-q)$
(c) $\frac{\log p}{\log q}$
(d) $\log\left(\frac{p}{q}\right)$

ANSWER KEY

1.	c	**2.**	b	**3.**	d	**4.**	a	**5.**	b
6.	a	**7.**	c	**8.**	a	**9.**	c	**10.**	b
11.	b	**12.**	a	**13.**	b	**14.**	a	**15.**	a
16.	a	**17.**	a	**18.**	a	**19.**	b	**20.**	a
21.	a	**22.**	a	**23.**	d				