Unit 08

LINEAR GRAPHS AND THEIR APPLICATION

An Ordered Pair of Real Numbers

An ordered pair of real numbers x and y is pair (x, y) in which elements are written in specific order. i.e.,

(i) (x,y) is an ordered pair in which first element is x and second is y such that $(x, y) \neq (y, x)$ for example:

(2, 3) and (3, 2) are two different ordered pairs.

(ii) (x, y) = (m, n) if and only if x = m and y = n.

Cartesian Plane

The Cartesian plane establishes one-to-one correspondence between the set of ordered pairs $R \times R = \{(x, y) \mid x, y \in R\}$ and the points of the Cartesian plane.

In plane two mutually perpendicular straight lines are drawn. The lines are called the coordinate axes. The point O, where the two lines meet is called origin. This plane is called the coordinate plane or the Cartesian plane.

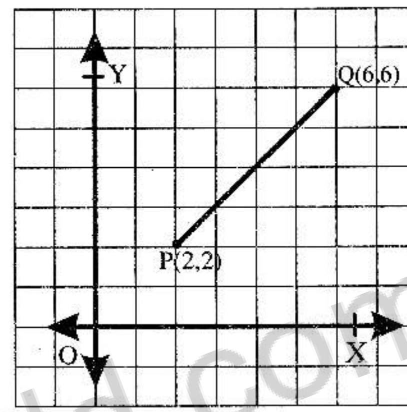
Drawing different geometrical Shapes in Cartesian Plane

(a) Line-Segment

Example:

Let P(2, 2) and Q(6, 6) be two points.

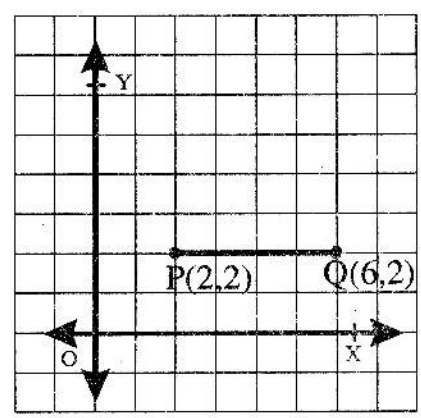
1. Plot points P and Q.
2. Join the points P and Q, we get the line segment PQ. It is represented by $\overline{PQ}$.



Example:

Plot points P(2, 2) and Q(6, 2). By joining them, we get a line segment PQ parallel to x-axis,

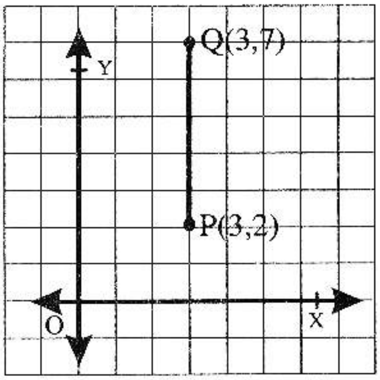
Where ordinate of both points is equal.



Example:

Plot points P(3, 2) and Q(3, 7). By joining them, we get a line segment PQ parallel to y-axis.

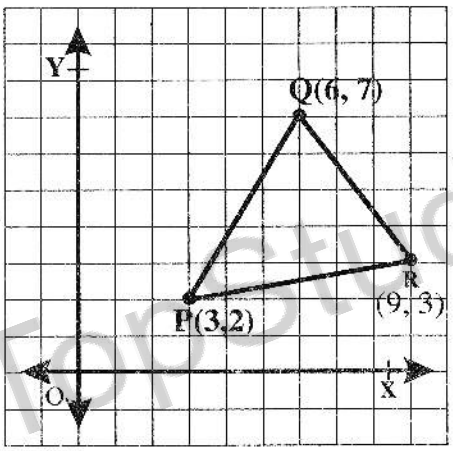
In this graph abscissas of both points are equal.



(b) Triangle

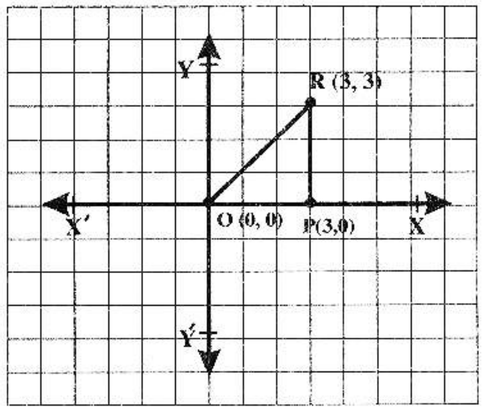
Example:

Plot the points P(3, 2), Q(6, 7) and R(9, 3). By joining them, we get a triangle PQR.



Example:

For points O(0, 0), P(3, 0) and R (3, 3), the triangle OPR is constructed.



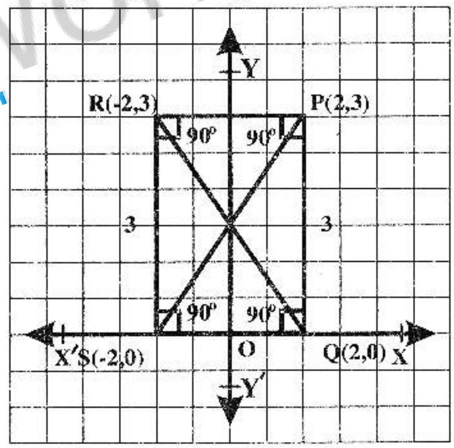
(c) Rectangle

Example

Plot the points P(2, 3), Q(2, 0), S(–2, 0) and R (–2, 3). Joining the points P, Q ,S and R, we get a rectangle PQSR.

Along y-axis,

2 (Length of square) = 1 unit



Construction of a Table for Pairs of Values Satisfying a Linear Equation in Two Variables.

Let $2x + y = 1$ (i)

Be a linear equation in two variables x and y.

The ordered pair (x, y) satisfies the equation and by varying x, corresponding y is obtained.

We express (i) in the form

$$\boxed{y = 1 - 2x} \qquad \text{(ii)}$$

The pairs (x, y) which satisfy (ii) are tabulated below.

x	y	(x, y)
−1	3	(−1, 3)
0	1	(0,1)
1	−1	(1,−1)
3	−5	(3, −5)

at $x=-1, y=(-2)(-1)+1=2+1=3$

at $x=0, y=(-2)(0)+1=0+1=1$

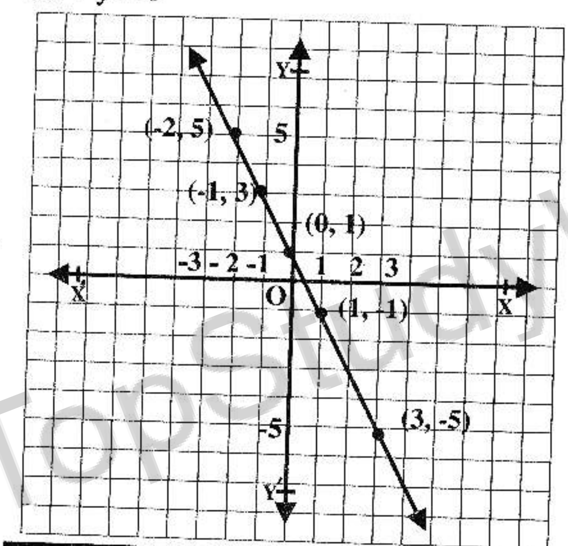
at $x=1, y=(-2)(1)+1=-2+1=-1$

at $x=3, y=-2(3)+1=-6+1=-5$

Similarly all the points can be computed, the ordered pairs of which do satisfy the equation (i)

Plotting the points to get the graph

Now we plot the points obtained in the table. Joining these points we get the graph of the equation. The graph of $2x + y = 1$



Example:

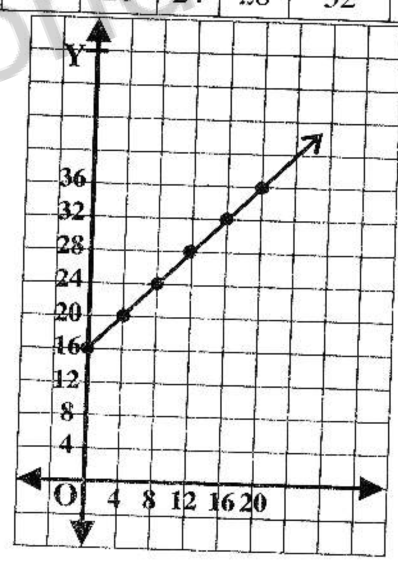
Equation $y = x + 16$ shows the relationship between the age of father and son i.e., if the age of son is x, then the father's age is y. Draw the graph.

Solution:

We know that $y = x + 16$

Table of points for equation is given as:

x	0	4	8	12	16	20
y	16	20	24	28	32	36



Exercise 8.1

1. Determine the quadrant of the coordinate plane in which the following points lie.

Ans. (i) P (−4, 3) II quadrant

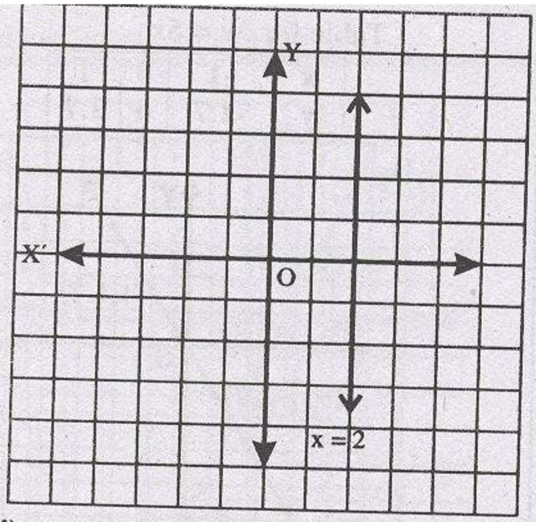
(ii) Q (−5, −2) III quadrant

(iii) P (2, 2) I quadrant

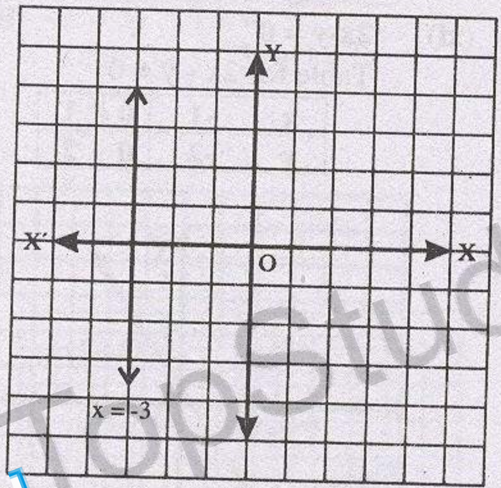
(iv) S(2, −6) IV quadrant

2. Draw the graph of each of the following.

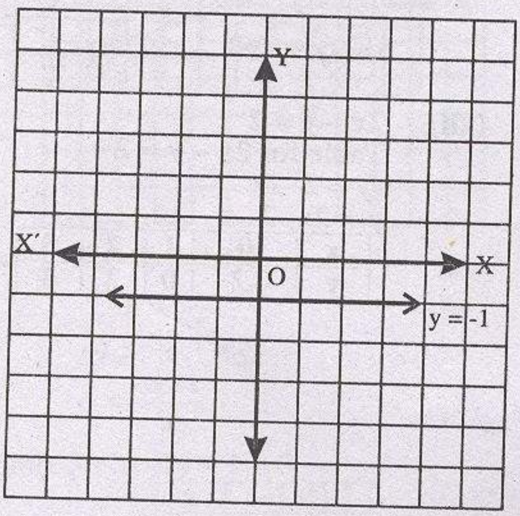
(i) $x=2$



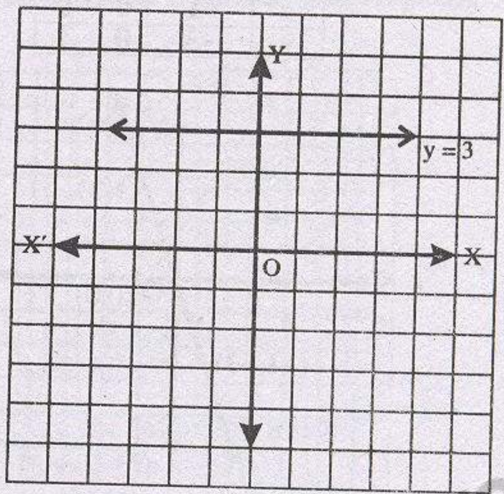
(ii) $x = -3$



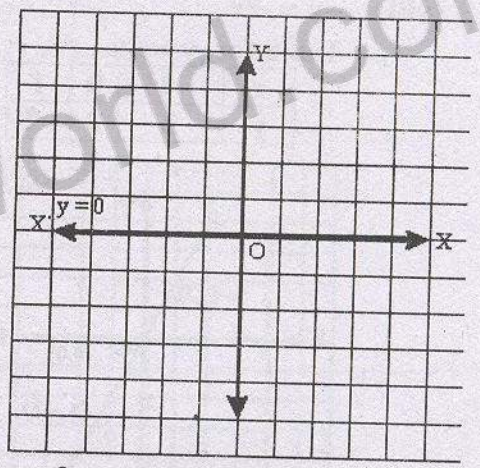
(iii) $y = -1$



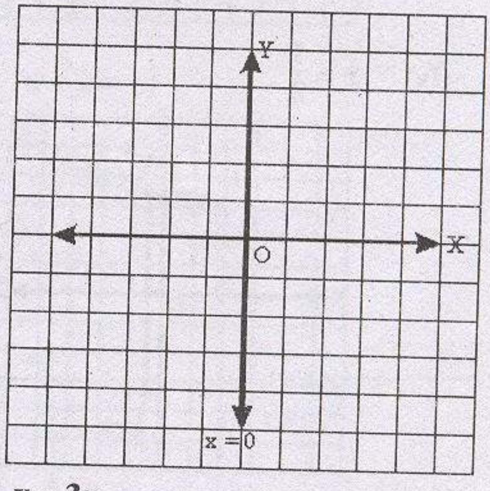
(iv) $y = 3$



(v) $y = 0$



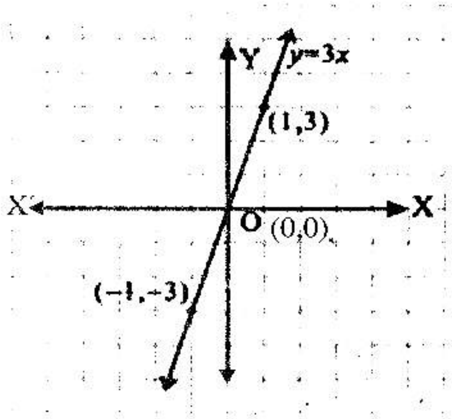
(vi) $x = 0$



(vii) $y = 3x$

Table for y = 3x

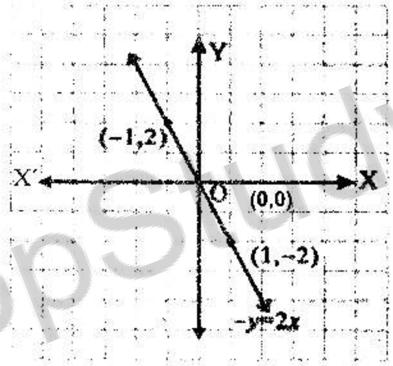
x	-1	0	1
y	-3	0	3



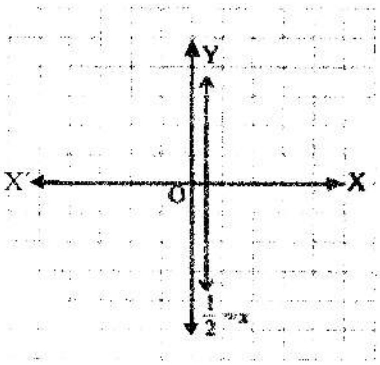
(viii) $-y = 2x$

Table for -y = 2x

x	-1	0	1
y	2	0	-2



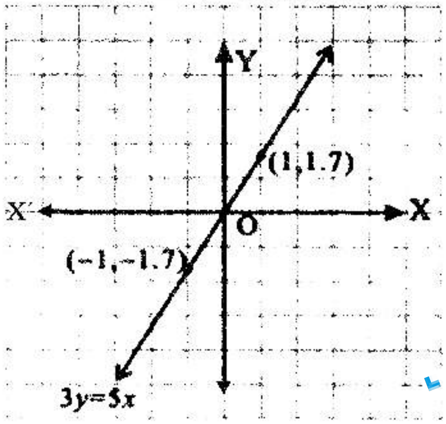
(ix) $x = \frac{1}{2}$



(x) $3y = 5x$

Table for 3y = 5x

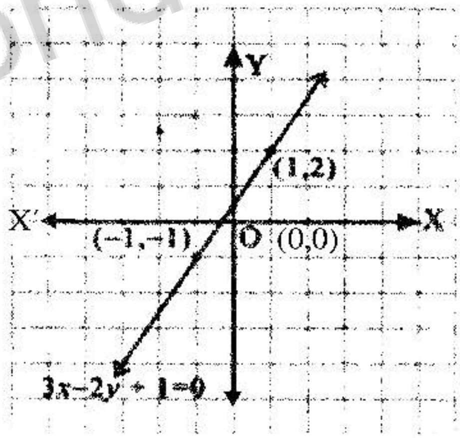
x	-1	0	1
y	-1.7	0	1.7



(xi) $2x - y = 0$

Table for 2x - y = 0

x	-1	0	1
y	-2	0	2



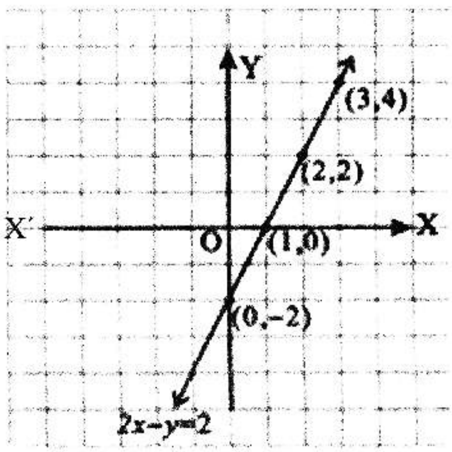
(xii) $2x - y = 2$

Table for 2x − y = 2

-y = 2 -2x

y = 2x -2

x	0	1	2	3
y	-2	0	2	4



(xiii) $x - 3y + 1 = 0$

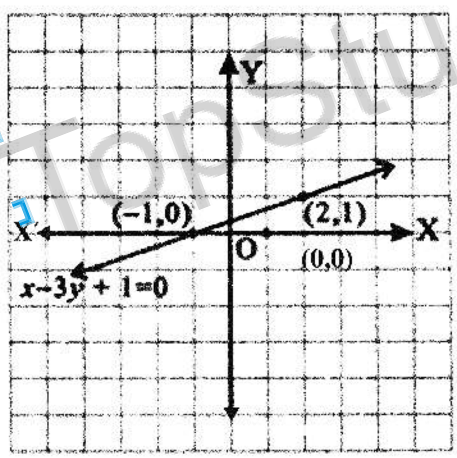
Table for $x - 3y + 1 = 0$

$-3y = -x - 1$

$3y = x + 1$

$y = \frac{x+1}{3}$

x	-1	2
y	0	1



(xiv) $3x - 2y + 1 = 0$

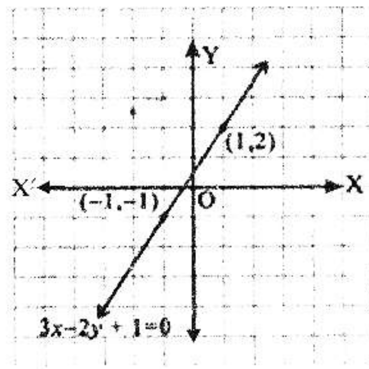
$-2y = -3x - 1$

$2y = 3x + 1$

$y = \frac{3x+1}{2}$

Table for $3x - 2y + 1 = 0$

x	-1	1
y	-1	2



Q.3 Are the following lines:

(i) Parallel to x–axis

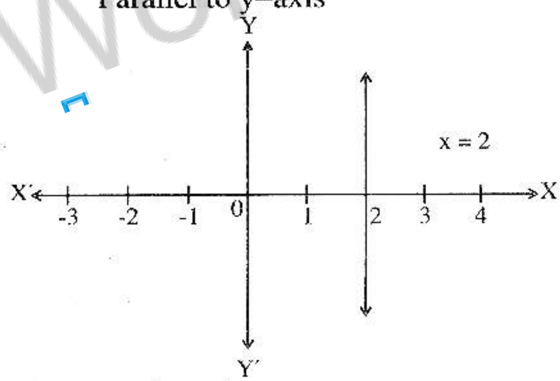
(ii) Parallel to y–axis

(i) $2x - 1 = 3$

$2x = 3 + 1$

$x = \frac{4}{2} = 2$

Parallel to y–axis

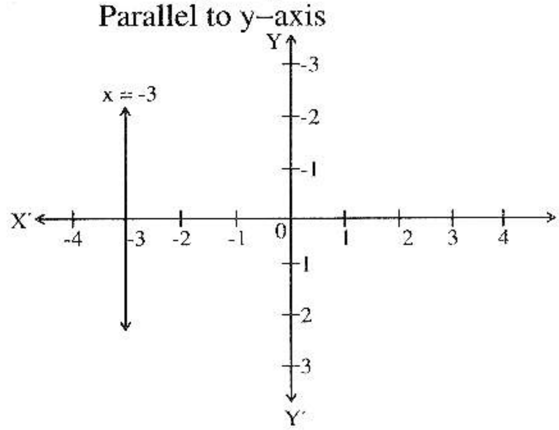


(ii) $x + 2 = -1$

$\Rightarrow \quad x = -1 - 2$

$x = -3$

Parallel to y–axis

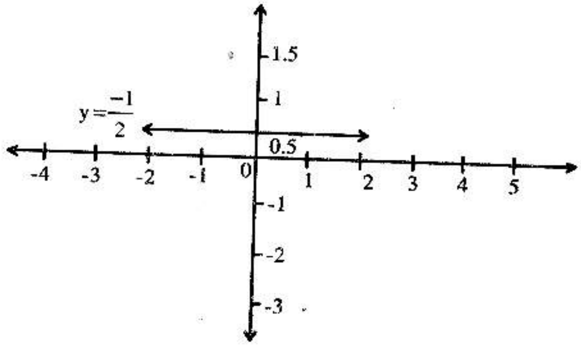


(iii) $2y + 3 = 2$

$\Rightarrow \quad 2y = 2 - 3$

$y = -\frac{1}{2}$

Parallel to x–axis



(iv) $x + y = 0$

$\Rightarrow \quad x = -y$

neither

(v) $2x - 2y = 0$

$2x = 2y$

$x = y$

neither

Q.4 Find the value of m and c of the following lines by expressing them in the form $y = mx + c$

(a) $\mathbf{x - 2y = -2}$

$-2y = -2 - x$

$2y = 2 + x$

$y = \frac{2+x}{2}$

$y = 1 + \frac{1}{2}x$

$y = \frac{1}{2}x + 1$ ....(1)

$y = mx + c$ .....(2)

comparing (1) and (2) we get

$m = \frac{1}{2}$ and $c = 1$

(b) $\mathbf{2x + 3y - 1 = 0}$

$3y = -2x + 1$

$y = \frac{-2x+1}{3}$

$y = \frac{-2}{3}x + \frac{1}{3}$ ....(1)

$y = mx + c$ ....(2)

comparing (1) and (2) we get

$m = \frac{-2}{3}$ and $c = \frac{1}{3}$

(c) $\mathbf{3x + y - 1 = 0}$

$y = -3x + 1$.....(1)

Also $y = mx + c$......(2)

Comparing (1) and (2)

$m = -3$ and $c = 1$

(d) $\mathbf{2x - y = 7}$

$-y = 7 - 2x$

$y = -7 + 2x$

$y = 2x - 7$.....(1)

also $y = mx + c$ ......(2)

comparing (1) and (2)

$m = 2$ and $c = -7$

(e) $\mathbf{3 - 2x + y = 0}$

$y = -3 + 2x$

$y = 2x - 3$ ....(1)

Also $y = mx + c$.....(2)

Comparing (1) and (2) we get

$m = 2$ and $c = -3$

(f) $\mathbf{2x = y + 3}$

$y = 2x - 3$ ....(1)

Also $y = mx + c$.....(2)

Comparing (1) and (2) we get

$m = 2$ and $c = -3$

Q.5 Verify whether the following points lies on the line $2x - y + 1 = 0$ or not.

Ans. $2x - y + 1 = 0$

(i) $(2, 3) \Rightarrow x = 2, y = 3$

$2x - y + 1 = 0$

$\Rightarrow 2(2) - 3 + 1 = 0$

$4 - 3 + 1 \neq 0$

$2 \neq 0$ Point (2,3) does not lie on the line

(ii) $(0, 0) \Rightarrow x = 0, y = 0$

$2x - y + 1 = 0$

$\Rightarrow 2(0) - 0 + 1 = 0$

$1 \neq 0$

Point (0,0) does not lie on the line

(iii) $(-1, 1) \Rightarrow x = -1, y = 1$

$2x - y + 1 = 0$

$\Rightarrow 2(-1) - (1) + 1 - 0 = 0$

$-2 - 1 + 1 = 0$

$-2 \neq 0$

Point (-1,1) does not lie on the line

(iv) $(2, 5) \Rightarrow x = 2, y = 5$

$2x - y + 1 = 0$

$\Rightarrow 2(2) - 5 + 1 = 0$

$4 - 5 + 1 = 0$

$-1 + 1 = 0$

$0 = 0$

Yes the Point (2,5) lies on the line

(v) $(5, 3) \Rightarrow x = 5, y = 3$

$2x - y + 1 = 0$

$\Rightarrow 2(5) - 3 + 1 = 0$

$10 - 2 = 0$

$8 \neq 0$

The point (5, 3) does not lie on the line

(a) Example: (Kilometre (Km) and Mile (M) Graphs)

To draw the graph between kilometre (Km) and Miles (M), we use the following relation:

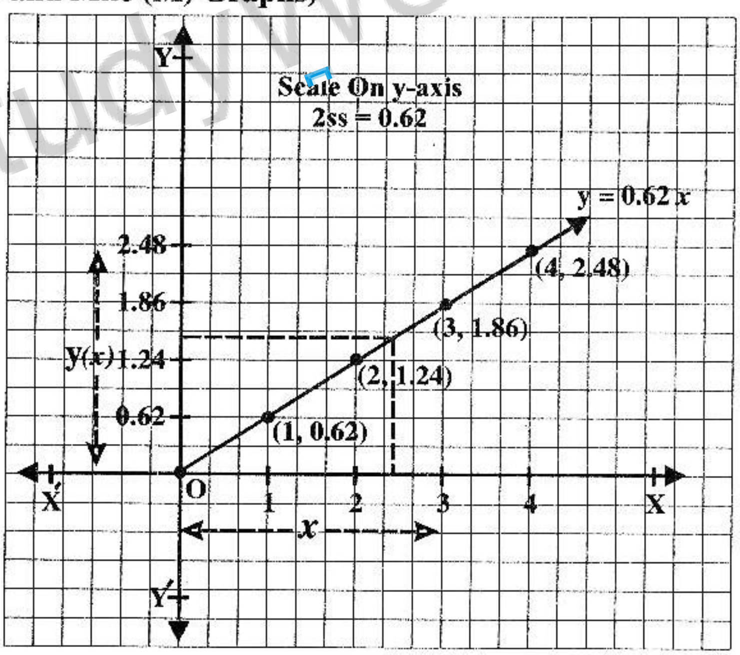
One kilometre = 0.62 miles, (approximately)

And One mile = 1.6 km (approximately)

(i) The relation of mile against kilometre is given by the linear equation,

$$y = 0.62\,x,$$

If y is a mile and x is a kilometre, then we tabulate the ordered pairs (x, y) as below;



x	0	1	2	3	4
y	0	0.62	1.24	1.86	2.48

The ordered pairs (x, y) corresponding to y = 0.62x are represented in the Cartesian plane. By joining them we get the desired graph of miles against kilometers.

For each quantity of kilometre x along x-axis their corresponding mile along y-axis.

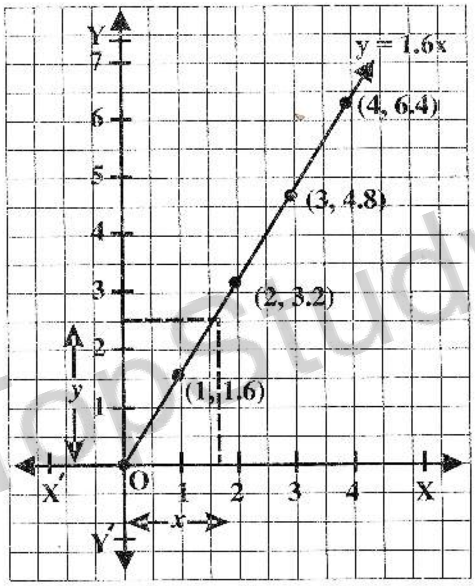
(ii) The conversion graph of kilometer against mile is given by

$y = 1.6x$ (approximately)

If y represents kilometers and x a mile, then the values x and y are tabulated as:

x	0	1	2	3	4 ....
y	0	1.6	3.2	4.8	6.4 ...

We plot the points in the xy-Plane corresponding to the ordered pairs. (0,0), (1, 1.6), (2, 3.2) (3, 4.8) and (4, 6.4) as shown in figure.



By joining the points we actually find the conversion graph of kilometers against miles.

(b) Conversion Graph of Hectares and Acres

(i) The relation between Hectare and Acre is defined as:

$$\text{Hectare} = \frac{640}{259}\text{ Acres}$$

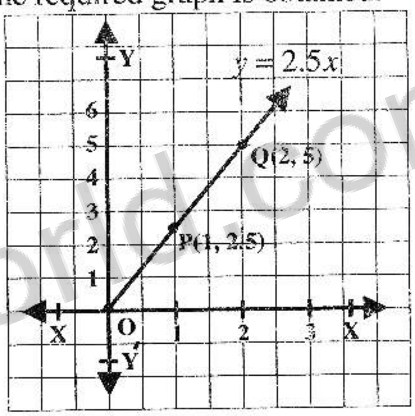
$= 2.5$ Acres (approximately)

In case when hectare = x and acre = y, then relation between them is given by the equation, $y = 2.5x$

If x is represented as hectare along the horizontal axis and y as Acre along y-axis, the values are tabulated below:

x	0	1	2	3	4 ....
y	0	2.5	5.0	7.5	10 ....

The ordered pairs (0, 0), (1, 2.5), (2,5) etc., are plotted as points in the xy-plane as below and by joining the points the required graph is obtained:



b – (i)

(ii) Now the conversion graph

$\text{Acre} = \frac{1}{2.5}$ Hectare is simplified as,

$$\text{Acre} = \frac{10}{25}\text{ Hectare}$$

$= 0.4$ Hectare (approximately)

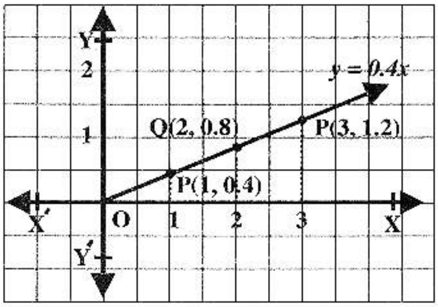
If Acre is measured along x-axis and hectare along y-axis then

$y = 0.4x$

The ordered pairs are tabulated in the following table:

x	0	1	2	3 ...
y	0	0.4	0.8	1.2 ...

The corresponding ordered pairs (0, 0), (1, 0.4), (2, 0.8) etc., are plotted in the xy-plane, join of which will form the graph of (b)-ii as a conversion graph of (b)-i:



b – (ii)

(c) Conversion Graph of Degrees Celsius and Degrees Fahrenheit

(i) The relation between Celsius (C) and degree Fahrenheit (F) is given by

$$F = \frac{9}{5}C + 32$$

The value of F at C = 0 is obtained as

$$F = \frac{9}{5} \times 0 + 32 = 0 + 32 = 32$$

Similarly,

$$F = \frac{9}{5} \times 10 + 32 = 18 + 32 = 50,$$

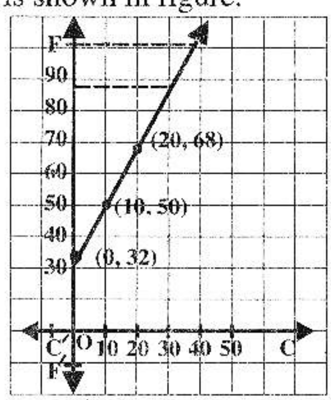
$$F = \frac{9}{5} \times 20 + 32 = 36 + 32 = 68,$$

$$F = \frac{9}{5} \times 100 + 32 = 180 + 32 = 212$$

We tabulate the values of C and F.

C	0°	10°	20°	50°	100°…
F	32°	50°	68°	122°	212°…

The conversion graph of F with respect to C is shown in figure.



10° = length of square

(d) Conversion graph of US$ and Pakistani Currency

The daily News, on a particular day informed the conversion rate of Pakistani currency to the US$ currency as.

1 US$ = 66.46 Rupees

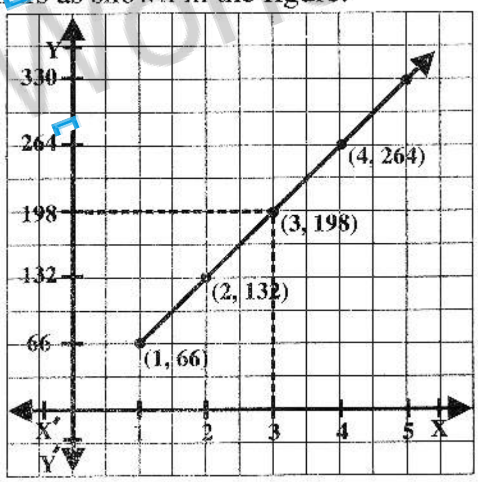
If the Pakistani currency y is an expression of US$ x, expressed under the rule

$y = 66.46\,x \cong 66x$ (approximately)

Then draw the conversion graph.

x	1	2	3	4 …
y	66	132	198	264 …

Plotting the points corresponding to the ordered pairs (x, y) from the above table and joining them provides the currency linear graph of rupees against dollars as shown in the figure.



Conversion graph $x = \frac{1}{66}y$ of y = 66x can be shown by interchanging x-axis to y-axis and vice versa.

Exercise 8.2

Q.1Draw the conversion graph between 1 litre and gallons using the relation 9 litres = 2 gallons (approximately) and taking litres along horizontal axis and gallons alongs vertical axis. From the graph, read:

(i) The number of gallons in 18 litres

(ii) The number of litres in 8 gallons

Ans.

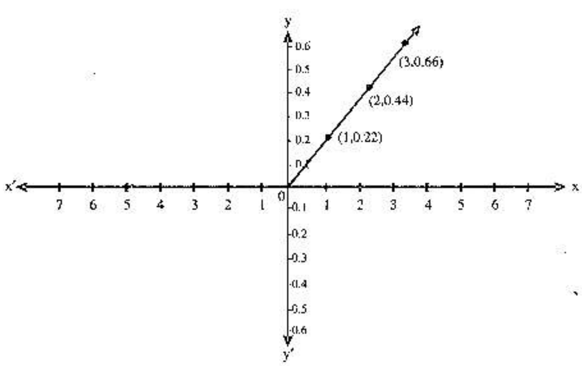
9 litres = 2 gallons	
1 litre = $\frac{2}{9}$ gallons	1 gallon = $\frac{9}{2}$ liter
1 litre= 0.222gallons	1 gallon=4.5 liter

Let gallon be represent by y and litre be x

$$y = 0.222x$$

Table of values

x	0	1	2	3
y	0	0.222	0.444	0.666



(i) Number of gallons in litre

$y = 0.222(18) = \; = 4$ gallons

(ii) Number of litres in 8 gallons

$\frac{9}{2}(8) = 36$ litres

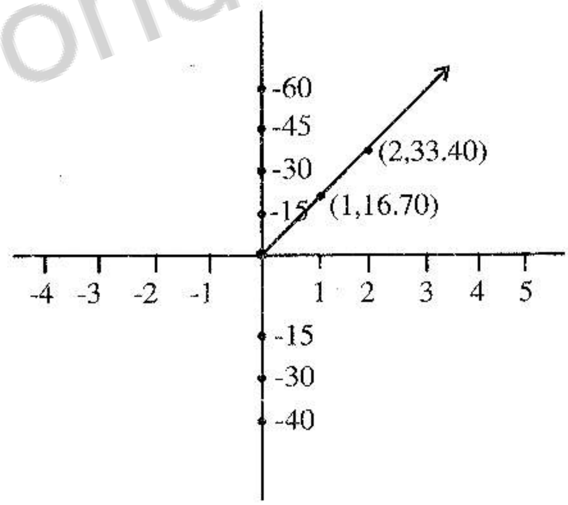
Q.2 On 15.03.2008 the exchange rate of Pakistani currency and Saudi Riyal was as, under 1 S. Riyal = 16.70 rupees.

If Pakistani currency y is an expression of S. Riyal x, expressed under the rule y = 16.70x then draw conversion graph between two currencies by taking S. Riyal along x–axis.

Ans. y = 16.70x.

Table of values

x	0	1	2
y	0	16.70	33.40



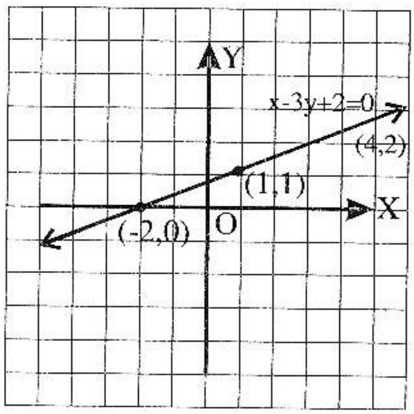
Q.3 Sketch the graph of each of the following lines:

Ans.

(i) $x - 3y + 2 = 0 \Rightarrow -3y = -x - 2$

$$y = \frac{x+2}{3}$$

x	−2	1	4
y	0	1	2



(ii) $3x - 2y - 1 = 0$

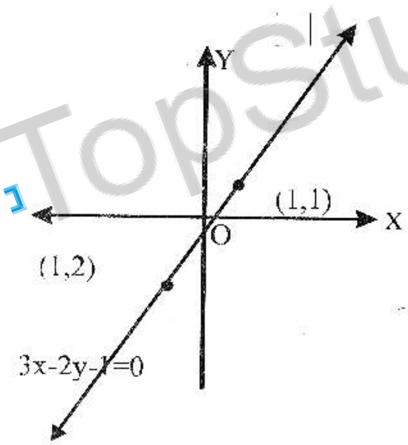
$-2y = 1 - 3x$

$2y = -1 + 3x$

$y = \frac{3x-1}{2}$

Table of values

x	–1	1	3
y	–2	1	4



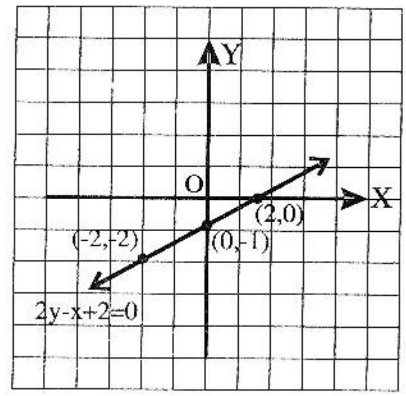
(iii) $2y - x + 2 = 0$

$2y = x - 2$

$y = \frac{x-2}{2}$

Table of values

x	–2	0	2
y	–2	–1	0

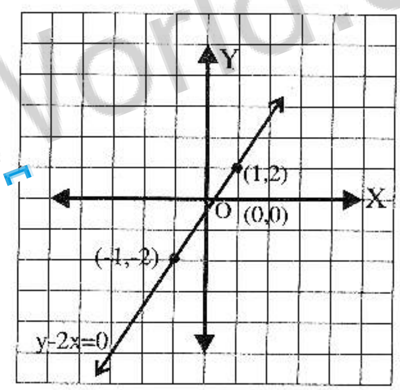


(iv) $y - 2x = 0$

$y = 2x$

Table of values

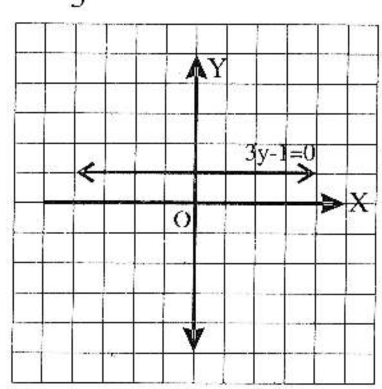
x	–1	0	1
y	–2	0	2



(v) $3y - 1 = 0$

$3y = 1$

$y = \frac{1}{3}$



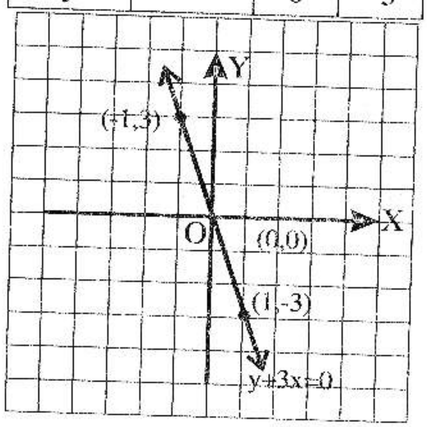
3 (length of square) = 1 unit

(vi) $y + 3x = 0$

$y = -3x$

Table of values

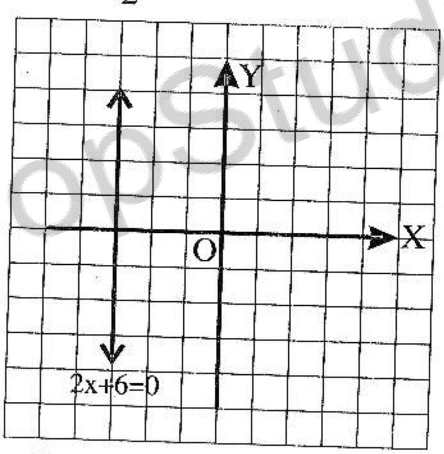
x	−1	0	1
y	3	0	−3



(vii) $2x + 6 = 0$

$2x = -6$

$x = \frac{-6}{2} = -3$



Q.4 **Draw the graph for following relations:**

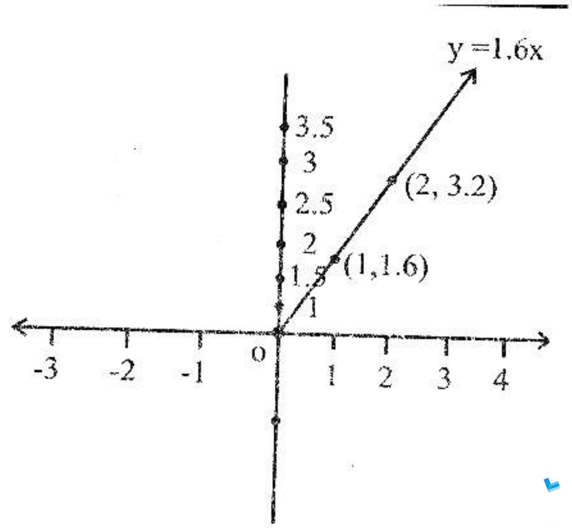
(i) **One mile = 1.6 km**

Let mile be represented by y and km by x:

$y = 1.6x$

Table of values

X	1	2	3
y	1.6	3.2	4.8

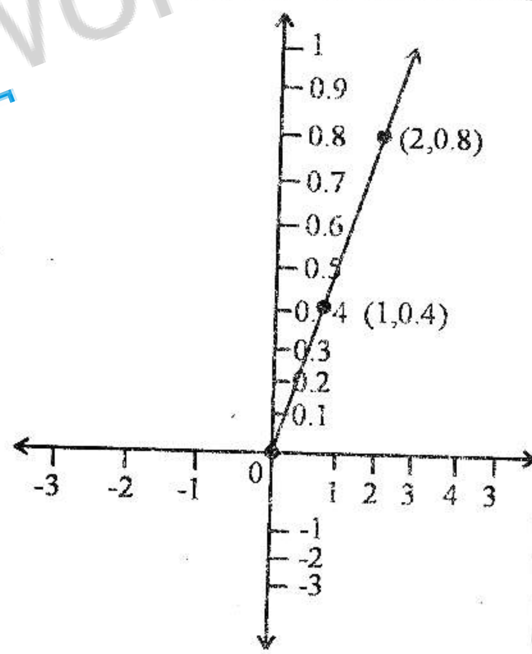


(ii) **One acre =0.4 Hectare**

$y = 0.4x$

Table of values

x	0	1	2
y	0	0.4	0.8



(iii) $F = \frac{9}{5}c + 32$

The value of F at C = 0 is obtained

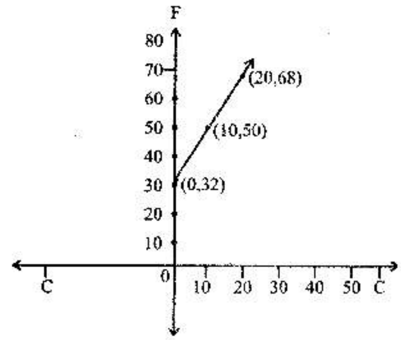
As $F = \frac{9}{5} \times 0 + 32 = 0 + 32 = 32$

$F=\frac{9}{5}\times 10+32=36+32=68$

$F=\frac{9}{5}\times 100+32=180+32=212$

We tabulate the values of C and F

C	0°	10°	20°	50°	100°
F	32°	50°	68°	122°	212°



Graphical Solution of Linear equations in Two Variables

We solve here simultaneous linear equations in two variables by graphical method

Let the system of equations be,

2x − y = 3, ...(i)

x + 3y = 3. ......(ii)

Table of Values

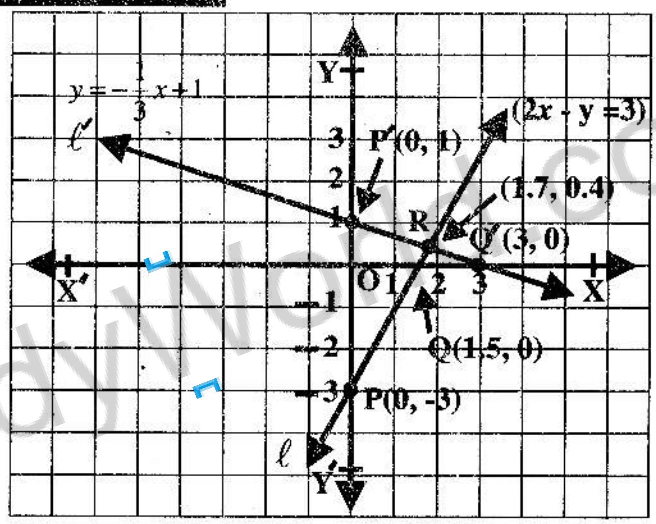
$y=2x-3$

x	0	1.5
y	−3	0

$y=-\frac{1}{3}x+1$

x	0	3
y	1	0

By plotting the points we get the following graph.



The solution of the system is the point R where the lines ℓ and ℓ' meet at, i.e., R(1.7,0.4) such that $x=1.7$ and $y=0.4$

Example

Solve graphically, the following linear system of two equations in two variables x and y;

$x+2y=3,$ ....(i)

$x-y=2.$ ....(ii)

Solution

The equations (i) and (ii) are represented graphically with the help of their points of intersection with the coordinate axes of the same co-ordinate plane.

The points of intersections of the lines representing equation (i) and (ii) are given in the following table:

$y=-\frac{1}{2}x+\frac{3}{2}$

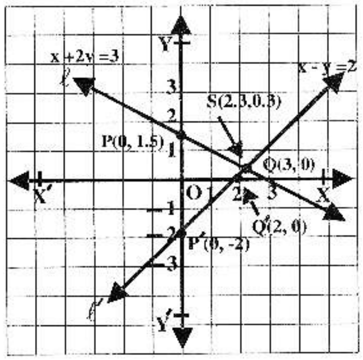
x	0	3
y	1.5	0

$y=x-2$

x	0	2
y	−2	0

The points P(0, 1.5) and Q (3, 0) of equation (i) are plotted in the plane and the corresponding line $\ell: x+2y=3$ is traced by joining P and Q.

Similarly, the line $\ell' : x - y = 2$ of (ii) is obtained by plotting the points $P'(0,-2)$ and $Q'(2,0)$ in the plane and joining them to trace the line ℓ' as below:



The common point S(2.3, 0.3) on both the lines ℓ and ℓ' is the required solution of the system.

Exercise 8.3

Solve the following pair of equations in x and y graphically.

Q.1 x + y = 0 and 2x – y + 3 = 0

Solution: ⇒ y = 0 – x

Table of values

x	–3	–2	–1	0	1	2
y	3	2	1	0	–1	–2

2x – y + 3 = 3

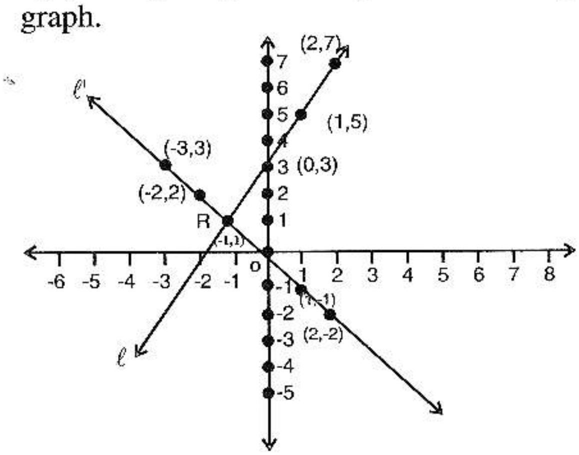
⇒ –y = –3 – 2x

y = 3 + 2x

Table of values

x	–2	–1	0	1	2
y	–1	1	3	5	7

By plotting the points we get the following graph.



The solution of the system is the point R where the lines ℓ and ℓ' meet at R(–1,1) such that x = –1 and y = 1

Q.2 x – y + 1 = 0 and x – 2y = –1

Solution: y = x + 1

Table of values,

x	–4	–3	–2	–1	0	1	2
y	-3	-2	-1	0	1	2	3

x – 2y = –1

–2y = –1 – x

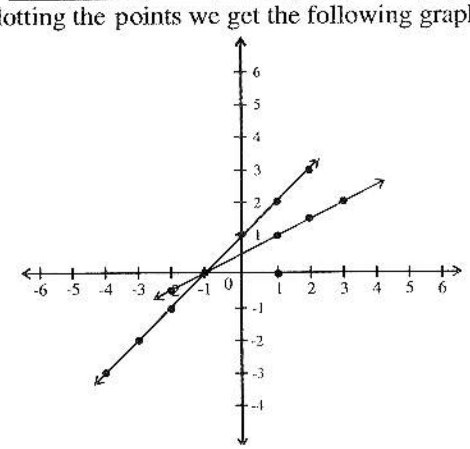
2y = 1 + x

$$y = \frac{1+x}{2}$$

Table of values,

x	–2	–1	0	1	2	3
y	–0.5	0	0.5	1	1.5	2

By plotting the points we get the following graph



The solution of the system is the point R where the lines ℓ and ℓ' meet at R (−1, 0) such that x = −1 and y = 0

Q.3 2x + y = 0 and x + 2y = 2

Solution: y = −2x

Table of the values

x	−2	−1	0	1	2	3	4
y	4	2	0	−2	−4	−6	−4

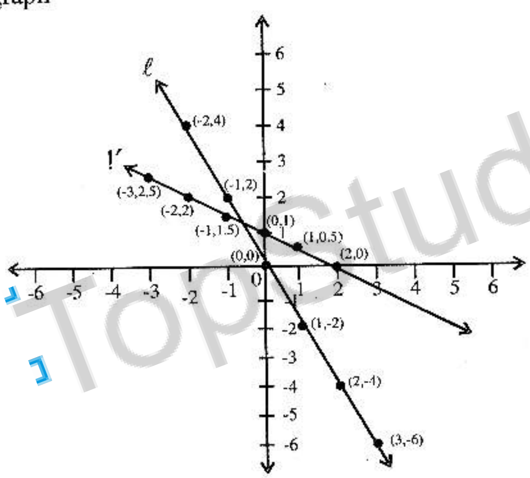
x + 2y = 2

2y = 2 − x

$$y = \frac{2-x}{2}$$

x	−3	−2	−1	0	1	2
y	2.5	2	1.5	1	0.5	0

By plotting the points we get the following graph



The solution of equations is $R\left(-\frac{2}{3}, \frac{4}{3}\right)$

Q.4 x + y − 1 = 0
x − y + 1 = 0

Solution: x + y = 1

y = 1 − x

Table of values

x	−3	−2	−1	0	1	2
y	4	3	2	1	0	−1

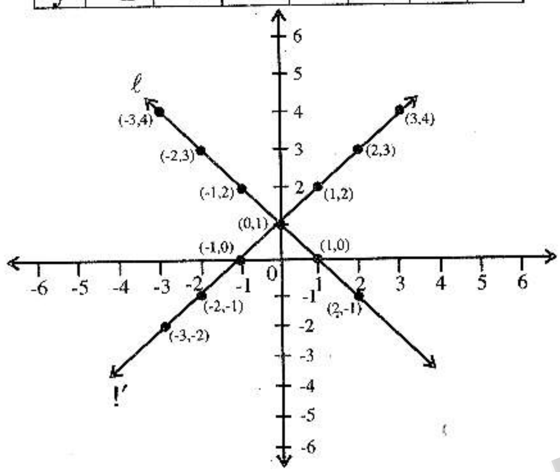
x − y + 1 = 0

−y = −1 − x

y = 1 + x

Table of values,

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4



The solution of the systems is R(0,1)

Q.5 2x + y − 1 = 0, x = −y

Solution: 2x + y = 1

y = 1 − 2x

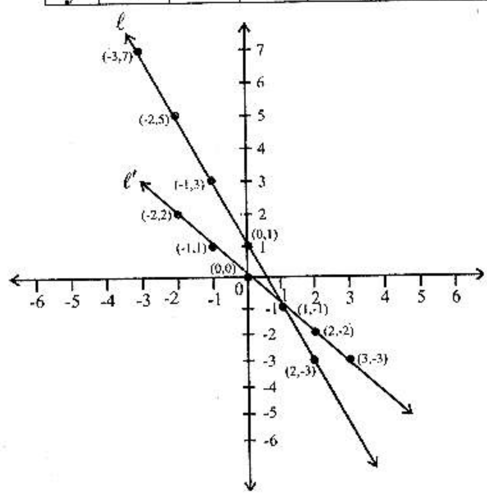
Table of values

x	−3	−2	−1	0	1	2
y	7	5	3	1	−1	−3

x = −y

Table of values

x	−2	−1	0	1	2	3
y	2	1	0	−1	−2	−3



The solution of the system is the point R where the lines ℓ and ℓ' meet at R(1, −1) such that x = 1 and y = −1.

Objective

1. If (x–1, y+1) = (0, 0), then (x, y) is:
   (a) (1, –1) (b) (–1, 1)
   (c) (1, 1) (d) (–1, –1)
2. If (x, 0) = (0, y), then (x, y) is:
   (a) (0, 1) (b) (1, 0)
   (c) (0, 0) (d) (1, 1)
3. Point (2 –3) lies in quadrant:
   (a) I (b) II
   (c) III (d) IV
4. Point (–3, –3) lies in quadrant:
   (a) I (b) II
   (c) III (d) IV
5. If y = 2x + 1, x = 2 then y is:
   (a) 2 (b) 3
   (c) 4 (d) 5
6. Which ordered pair satisfy the equation y = 2x:
   (a) (1, 2) (b) (2, 1)
   (c) (2, 2) (d) (0, 1)
7. The real numbers x, y of the ordered pair (x, y) are called ______ of point P(x,y) in a plane:
   (a) co-ordinates
   (b) x co-ordinates
   (c) y-coordinate
   (d) ordinate
8. Cartesian plane is divided into __ quadrants:
   (a) Two (b) Three
   (c) Four (d) Five
9. The point of intersection of two coordinate axes is called:
   (a) Origin (b) Centre
   (c) X-coordinate (d) y-coordinate
10. The x-coordinate of a point is called__
    (a) Origin (b) abcissa
    (c) y-coordinate (d) Ordinate
11. The y-coordinate of a point is called:
    (a) Origin (b) x-coordinate
    (c) y-coordinate (d) ordinate
12. The set of points which lie on the same line are called ___ points:
    (a) Collinear (b) Similar
    (c) Common (d) None of these
13. The plane formed by two straight lines perpendicular to each other is called ___:
    (a) Cartesian plane
    (b) Coordinate axes
    (c) Plane
    (d) None of these
14. An ordered pair is a pair of elements in which elements are written in specific:
    (a) Order (b) Array
    (c) Point (d) None

Answer key

1.	a	**2.**	c	**3.**	d	**4.**	c	**5.**	d
6.	a	**7.**	a	**8.**	c	**9.**	a	**10.**	b
11.	d	**12.**	a	**13.**	a	**14.**	a		