Instructions: Complete each of the following on separate, stapled sheets of paper.

1. Prove that the graph $K_{m, n}$ has $m n$ edges.
2. What is the smallest number of edges that must be removed from $K_{5}$ to make a bipartite graph?
3. For each of the graphs $G$ below, compute the chromatic number $\chi(G)$. Give a complete proof.
(a)

(b)

4. Prove that every finite simple graph $G$ has at least $\binom{\chi(G)}{2}$ edges (where $\chi(G)$ is the chromatic number of $G$ ).
5. Let $G$ be a graph and let $\sim$ be the relation on $V(G)$ defined by $u \sim v$ when there is a walk in $G$ from $u$ to $v$. Prove that $\sim$ is an equivalence relation.
