Basic Logical Equivalences

On route to learning how to write proofs, we analyze the algebra of propositions. The following are fundamental logical equivalences. The names given here are nonstandard; we often refer to them by shorter names.

The symbols $\mathbb{A}$, $\mathbb{B}$, and $\mathbb{C}$ denote arbitrary statements, and T and F denote "True" and "False" respectively.

- $\neg(\neg\mathbb{A}) \equiv \mathbb{A}$ (Double Negation)
- $\mathbb{A} \vee (\neg\mathbb{A}) \equiv \mathrm{T}$ (Tautology)
- $\mathbb{A} \wedge (\neg\mathbb{A}) \equiv \mathrm{F}$ (Contradiction)
- $\mathbb{A} \vee \mathbb{A} \equiv \mathbb{A}$ ($\vee$-Idempotence)
- $\mathbb{A} \wedge \mathbb{A} \equiv \mathbb{A}$ ($\wedge$-Idempotence)
- $\mathbb{A} \vee \mathrm{T} \equiv \mathrm{T}$ ($\vee$-Definition)
- $\mathbb{A} \vee \mathrm{F} \equiv \mathbb{A}$ ($\vee$-Definition)
- $\mathbb{A} \wedge \mathrm{T} \equiv \mathbb{A}$ ($\wedge$-Definition)
- $\mathbb{A} \wedge \mathrm{F} \equiv \mathrm{F}$ ($\wedge$-Definition)
- $\mathbb{A} \vee \mathbb{B} \equiv \mathbb{B} \vee \mathbb{A}$ ($\vee$-Commutativity)
- $\mathbb{A} \wedge \mathbb{B} \equiv \mathbb{B} \wedge \mathbb{A}$ ($\wedge$-Commutativity)
- $\mathbb{A} \vee (\mathbb{B} \vee \mathbb{C}) \equiv (\mathbb{A} \vee \mathbb{B}) \vee \mathbb{C}$ ($\vee$-Associativity)
- $\mathbb{A} \wedge (\mathbb{B} \wedge \mathbb{C}) \equiv (\mathbb{A} \wedge \mathbb{B}) \wedge \mathbb{C}$ ($\wedge$-Associativity)
- $\mathbb{A} \vee (\mathbb{B} \wedge \mathbb{C}) \equiv (\mathbb{A} \vee \mathbb{B}) \wedge (\mathbb{A} \vee \mathbb{C})$ ($\vee\wedge$-Distribution)
- $\mathbb{A} \wedge (\mathbb{B} \vee \mathbb{C}) \equiv (\mathbb{A} \wedge \mathbb{B}) \vee (\mathbb{A} \wedge \mathbb{C})$ ($\wedge\vee$-Distribution)
- $\neg(\mathbb{A} \vee \mathbb{B}) \equiv (\neg\mathbb{A}) \wedge (\neg\mathbb{B})$ ($\vee$-DeMorgan Law)
- $\neg(\mathbb{A} \wedge \mathbb{B}) \equiv (\neg\mathbb{A}) \vee (\neg\mathbb{B})$ ($\wedge$-DeMorgan Law)
- $\mathbb{A} \oplus \mathbb{B} \equiv (\mathbb{A} \vee \mathbb{B}) \wedge (\neg(\mathbb{A} \wedge \mathbb{B}))$ ($\oplus$-Definition)
- $\mathbb{A} \implies \mathbb{B} \equiv (\neg\mathbb{A}) \vee \mathbb{B}$ (Material Implication)
- $\mathbb{A} \implies \mathbb{B} \equiv (\neg\mathbb{B}) \implies (\neg\mathbb{A})$ (Contraposition)
- $\mathbb{A} \iff \mathbb{B} \equiv (\mathbb{A} \implies \mathbb{B}) \wedge (\mathbb{B} \implies \mathbb{A})$ (Biconditional Expansion)