Practice: Propositional Logic 2

Instructions: Complete each of the following as practice.

1. Decide which statements below are valid, which are satisfiable, and which are invalid.[1]
   (a) $\neg(\neg P)$
   (b) $(P \implies Q) \implies (Q \implies P)$
   (c) $(Q \iff (\neg P)) \vee P$
   (d) $((P \implies Q) \wedge P) \implies Q$
   (e) $(P \vee Q) \implies (P \implies Q)$
   (f) $(P \implies Q) \implies Q$
   (g) $(P \oplus Q) \wedge (P \iff Q)$
   (h) $(P \implies Q) \iff ((\neg P) \vee Q)$
   (i) $((P \implies Q) \wedge (\neg P)) \implies (\neg Q)$
   (j) $(P \vee (\neg P)) \oplus (P \implies Q)$
   (k) $P \implies (Q \vee R)$
   (l) $((P \implies Q) \wedge (Q \implies R)) \iff (P \implies R)$
   (m) $((P \oplus Q) \oplus R) \iff (P \oplus (Q \oplus R))$
   (n) $(\neg(P \vee Q)) \wedge (R \implies P)$
   (o) $((P \implies Q) \implies (Q \implies R)) \implies (P \implies R)$

2. Compute the disjunctive normal form for each of the following statements.
   (a) $A \vee B$
   (b) $A \iff (B \vee (A \wedge (\neg B)))$
   (c) $A \vee (B \implies (\neg A))$
   (d) $(A \implies (B \vee (\neg A))) \implies A$
   (e) $(B \implies A) \wedge (B \oplus A)$
   (f) $(A \wedge (B \oplus C)) \iff A$
   (g) $(A \vee B) \vee ((\neg C) \implies (A \wedge B))$

3. Compute the conjunctive normal form for each of the following statements.
   (a) $A \wedge B$
   (b) $A \iff (B \vee (A \wedge (\neg B)))$
   (c) $A \vee (B \implies (\neg A))$
   (d) $(A \implies (B \vee (\neg A))) \implies A$
   (e) $(B \implies A) \wedge (B \oplus A)$
   (f) $(A \wedge (B \oplus C)) \iff A$
   (g) $(A \vee B) \vee ((\neg C) \implies (A \wedge B))$

4. Use the basic logical equivalences to prove the following (via the algebra of statements).
   (a) $A \iff B \equiv B \iff A$
   (b) $(\neg A) \wedge (A \vee B) \equiv B$
   (c) $(A \implies B) \wedge (\neg B) \equiv \neg(A \vee B)$

[1] HINT: You already computed truth tables for these statements in the first set of practice problems...